Department
of
Mathematics

Department of Mathematics

Summary Mathematics, originally centered in the concepts of number, magnitude, and form, has long been growing since ancient Egyptian times to the 21st century. Through the use of abstraction and logical reasoning, it became an indispensable tool not only in natural sciences, but also in engineering and social sciences. Recently, the remarkable development of computers is now making an epoch in the history of mathematics.

Department of Mathematics is one of the six departments of Graduate School of Science, Osaka University. It consists of 6 research groups, all of which are actively involved in the latest developments of mathematics. Our mathematics department has ranked among the top seven in the country.

The department offers a program with 32 new students enrolled annually leading to post-graduate degrees of Masters of Science. The department also offers a Ph.D. program with possibly 16 new students enrolled annually.

Graduate courses are prepared so as to meet various demands of students. Besides introductory courses for first year students, a number of topic courses are given for advanced students. Students learn more specialized topics from seminars under the guidance of thesis advisors.

Our department has our own library equipped with about 500 academic journals and 50,000 books in mathematics, both of which graduate students can use freely. In addition, by an online system, students as well as faculty members can look up references through Internet.

Research Groups

Algebra, Geometry, Analysis, Global Geometry & Analysis, Experimental Mathematics, Mathematical Science.

Areas of Research

Number Theory, Ring Theory, Algebraic Geometry, Algebraic Analysis, Partial Differential Equations, Real Analysis, Differential Geometry, Complex Differential Geometry, Topology, Knot Theory, Discrete Subgroups, Transformation Groups, Complex Analysis, Complex Functions of Several Variables, Complex Manifolds, Discrete Mathematics, Probability Theory, Dynamical Systems, Fractals, Mathematical Engineering, Information Geometry, Mathematical Physics.

Faculty Members

Professors (15)

Shin-ichi DOI, Akio FUJIWARA, Ryushi GOTO,
Masashi ISHIDA, Seiichi KAMADA,
Soichiro KATAYAMA, Hiroaki NAKAMURA,
Shin-ichi OHTA, Atsushi TAKAHASHI,
Naohito TOMITA, Takao WATANABE,
Katsutoshi YAMANOI, Kouji YANO,
Takehiko YASUDA, Masahiko YOSHINAGA.

Associate Professors (13)

Shinpei BABA, Kento FUJITA, Takahisa INUI,
Hisashi KASUYA, Yoshihiko MATSUMOTO,
Haruya MIZUTANI, Tomonori MORIYAMA,
Takahiro OBA, Mamoru OKAMOTO,
Shinnosuke OKAWA, Kazuto OTA,
Shuichiro TAKEDA, Motoo UCHIDA.

Lecturer (1)

Kazunori KIKUCHI.

Assistant Professors (9)

Yasuhiro HARA, Takao IOHARA, Masataka IWAI,
Kohei KIKUTA, Erika KUNO, Shohei
NAKAMURA,
Hiroyuki OGAWA, Koji OHNO, Toru SERA.

Cooperative Members in Osaka University

Professors (5)

Daisuke FURIHATA, Eiko KIN, Katsuhisa
MIMACHI,
Makoto NAKAMURA, Yoshie SUGIYAMA.

Associate Professors (6)

Tsuyoshi CHAWANYA, Akihiro HIGASHITANI,
Yuto MIYATAKE, Norio NAWATA,
Yasuhiro WAKABAYASHI, Kouichi YASUI.

Home Page

http://www.math.sci.osaka-u.ac.jp/eng/

Department of Mathematics

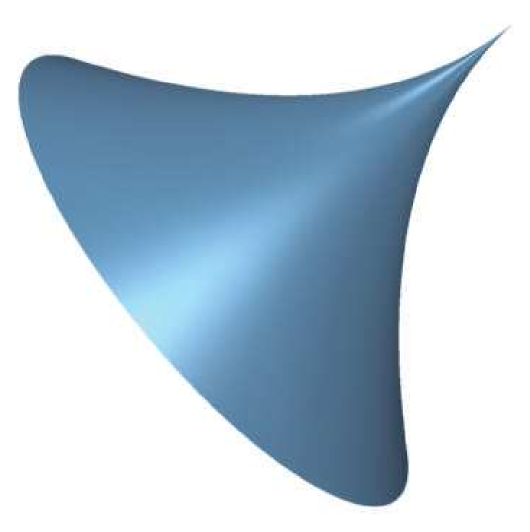
Shinpei BABA

Low-dimensional Geometry, Topology

My research is centered on surfaces (i.e. 2-dimensional manifolds), which are fundamental object in geometry and topology.

In particular, I am interested in the relations between geometric structures (locally homogeneous structures) on surfaces, and representations of the fundamental groups of surfaces S (surface groups) into Lie groups G.

In the case that the Lie groups G = PSL(2, R) or PSL(2,C) and that representations have discrete images, beautiful theories have been developed extensively, in particular, in relation to the classification and the deformation theorem of 2- and 3-dimensional manifolds.



Department of Mathematics

Shin-ichi DOI

Partial Differential Equations

Partial differential equations have their origins in various fields such as mathematical physics, differential geometry, and technology. Among them I am particularly interested in the partial differential equations that describe wave propagation phenomena: hyperbolic equations and dispersive equations. A typical example of the former is the wave equation, and that of the latter is the Schroedinger equation. For many years I have studied basic problems for these equations: existence and uniqueness of solutions, structure of singularities of solutions, asymptotic behavior of solutions, and spectral properties. Recently I make efforts to understand how the singularities of solutions for Schroedinger equations or, more generally, dispersive equations propagate. The center of this problem is to determine when and how the singularities of solutions for the dispersive equations can be described by the asymptotic behavior of solutions for the associated canonical equations.

Department
of
Mathematics

Kento FUJITA

Algebraic Geometry

My research interest is algebraic geometry. Especially, I am interested in birational behavior of Fano varieties, a special class of algebraic varieties. I have researched the Mukai conjecture, the existence of certain good models over some reducible varieties, and an algorithm to classify log del Pezzo surfaces (joint work with Kazunori Yasutake), etc. Recently, I am interested in K-stability of Fano varieties; I (and independently Chi Li) gave a birational interpretation of K-stability of Fano varieties.

Department
of
Mathematics

Akio FUJIWARA

Mathematical Engineering

"What is information?" Having this naive yet profound question in mind, I have been working mainly on noncommutative statistics, information geometry, quantum information theory, and algorithmic randomness theory.

One can regard quantum theory as a noncommutative extension of the classical probability theory. Likewise, quantum statistics is a noncommutative extension of the classical statistics. It aims at finding the best strategy for identifying an unknown quantum object from a statistical point of view, and is one of the most exciting research field in quantum information science.

Probability theory is usually regarded as a branch of analysis. Yet it is also possible to investigate the space of probability measures from a differential geometrical point of view. Information geometry deals with a pair of affine connections that are mutually dual (conjugate) with respect to a Riemannian metric on a statistical manifold. It is known that geometrical methods provide us with useful guiding principle as well as insightful intuition in classical statistics. I am interested in extending information geometrical structure to the quantum domain, admitting an operational interpretation.

I am also delving into algorithmic and game-theoretic randomness from an information geometrical point of view. Someday I wish to reformulate thermal/statistical physics in terms of algorithmic information theory.

Department of Mathematics

Ryushi GOTO

Geometry

My research interest is mostly in complex and differential geometry, which are closely related with algebraic geometry and theoretical physics. My own research started with special geometric structures such as Calabi-Yau, hyperKaehler, G2 and Spin(7) structures. These four structures exactly correspond to special holonomy groups which give rise to Ricci-flat Einstein metrics on manifolds. It is intriguing that these moduli spaces are smooth manifolds on which local Torelli type theorem holds. In order to understand these phenomenon, I introduce a notion of geometric structures defined by a system of closed differential forms and establish a criterion of unobstructed deformations of structures.

When we apply this approach to Calabi-Yau, hyperKaehler, G2 and Spin(7) structures, we obtain a unified construction of these moduli spaces. At present I also explore other interesting geometric structures and their moduli spaces.

Department of Mathematics

Yasuhiro HARA

Topology

The field of my study is topology and, especially, I study the theory of transformation groups. The Borsuk-Ulam theorem is one of famous theorems about transformation groups. This theorem is often taken up as an application in elementary lectures about the homology theory. The content of the theorem is as follows: for every continuous map from the n-dimensional sphere to the n-dimensional Euclidian space, there exists a point such that the map takes the same value at the point and at the antipodal point. A famous application of this theorem is the following. "On the earth, there is a point such that the temperature and humidity at the point are the same as those at the antipodal point." We consider a free action of a group of order two on the n-dimensional sphere to prove the Borsuk-Ulam theorem. Then for any equivariant map (any continuous map which preserves the structure of the group action) from the sphere to itself, the degree of the map is odd. By using this fact, we obtain the Borsuk-Ulam theorem. In the case of the Borsuk-Ulam theorem, we consider spheres and free actions of a group of order two. Actually, when we consider other manifolds and actions of other groups, there are some restrictions of homotopy types of equivariant maps. I study such restrictions of homotopy types of equivariant maps by using the cohomology theory, and I study relationships between homotopy types of equivariant maps and topological invariants.

Department of Mathematics

Takahisa INUI

Nonlinear Partial Differential Equations

My research subject is nonlinear partial differential equations. Especially, I am interested in the global behavior of the solutions to nonlinear dispersive equations or nonlinear wave equations.

Dispersive equations describe the dispersion phenomena of waves and are basic equations in quantum physics. For example, Schroedinger equation and Klein--Gordon equation are typical dispersive equations,

which appear in quantum physics or relativistic quantum field theory. Wave equations express the properties of motion in waves. Considering nonlinear interactions between particles, we can treat various physical phenomena, for example, optics, superconductivity, and Bose-Einstein condensate, by the nonlinear dispersive or wave equations. Nonlinear dispersive or wave equations have two properties.

One is dispersion and the other is nonlinearity. They conflict each other. Thus, there are so many behaviors of the solutions. I study this subject to find all behavior.

Department of Mathematics

Takao IOHARA

Nonlinear Partial Differential Equations

My research interest is concerned with nonlinear partial differential equations appearing in fluid mechanics. The current research topic is the equations of the motion of viscous incompressible fluid which has free moving surface. The motion of viscous incompressible fluid is governed by the Navier-Stokes equations, which are not easy to solve because of their nonlinearity. The free moving surface adds another nonlinearity to the problem and the study of it needs more elaborate technique than the problems on fixed domain.

Department of Mathematics

Masashi ISHIDA

Differential Geometry

My research interest is in geometry, particularly, interaction between topology and differential geometry. For instance, I am studying the nonexistence problems of Einstein metrics and Ricci flow solutions on 4-manifolds by using the Seiberg-Witten equations. I am also interested in the geometry of the Yamabe invariant. The computation of the Yamabe invariant for a given manifold is a difficult problem in general. By using the Seiberg-Witten equations, I determined the exact value of the Yamabe invariant for a large class of 4-manifolds which includes complex surfaces as special cases. Furthermore, I am also interested in both the Ricci flow in higher dimension and some generalized versions of the Ricci flow like the Ricci Yang-Mills flow.

The Ricci flow was first introduced by R. Hamilton in 1981 and used as the main tool in G. Perelman's solution of the Poincare conjecture in 2002. Perelman introduced many new and remarkable ideas to prove the conjecture. The theory developed by Hamilton and Perelman is now called the Hamilton-Perelman theory. One of my recent interests is to investigate geometric analytical properties of the generalized versions of the Ricci flow from the Hamilton-Perelman theoretical point of view.

Department of Mathematics

Masataka IWAI

Complex Geometry, Algebraic Geometry, Several Complex Variables

My main research object is a projective manifold, which is a submanifold of the complex projective space. Especially, I study the structure of projective manifolds whose tangent bundles or cotangent bundles have semipositive curvature. I have researched the structure of projective manifolds whose tangent bundles are semipositive by using the theory of singular Hermitian metrics. Finally, I established the structure theorem of semipositive foliations. Now, I'm studying the structure of projective manifolds whose cotangent bundles are semipositive.

The interesting part of studying projective manifolds is that we can use any techniques, such as complex geometry, algebraic geometry, several complex variables, and so on. Because projective manifolds are complex manifolds by definition, we can use the techniques of complex geometry and differential geometry. On the other hand, since projective manifolds can be represented by a zero set of polynomials by Chow's theorem, we can use the techniques of algebraic geometry and birational geometry. Furthermore, projective manifolds are built from a collection of unit balls in the complex Euclid space, we can use the techniques of several complex variables. In this way, I study projective manifolds using methods in a wide range of fields.

Department of Mathematics

Seiichi KAMADA

Topology

My research area is topology (geometric topology-- knot theory, low dimensional topology). I have been working on surface-links in 4-space, including 2-knots and 2-links, 2-dimensional braids, braided surfaces, and 4-manifolds.

Surface-links are closed surfaces smoothly embedded in the 4-space, and they are considered equivalent if they are ambiently isotopic. It is a quite difficult problem to decide whether two given surface-links are equivalent or not. Invariants are often used to show that two surface-links are not equivalent. However, few invariants are known for surface-links. I am interested in studying surface-links and their invariants using "quandles" and graphics so-called "charts".

A quandle is an algebra with a binary operation which satisfies three axioms corresponding to Reidemeister moves in knot theory. A homology theory of quandles was established and we can use it for construction of invariants for surface-links. 2-dimensional braids, or surface braids, are a generalization of classical braids. They are in one-to-one correspondence to oriented surface-links in 4-space modulo certain basics moves. As a method of describing 2-dimensional braids, a chart description was introduced. This method has been extended in a general theory of charts. For example, charts can be used for studies of Lefschets fibrations of 4-manifolds.

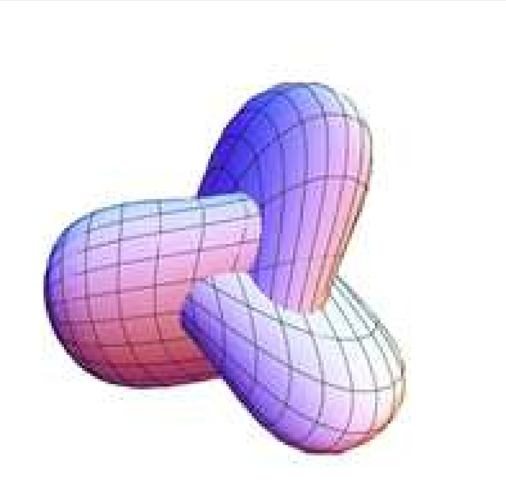
Department of Mathematics

Hisashi KASUYA

Geometry

Until now, I have tried to extend the geometry of nilpotent groups to the geometry of solvable groups. More precisely, I have studied the cohomology theory of homogeneous spaces of solvable Lie groups and complex geometry of non-Kähler manifolds. It seems that the gap between nilpotent groups and solvable groups is small. But this gap contains a potential for geometry. By the growing out of left-invariance and non-triviality of local system cohomology, I succeeded in giving a great surprise.

Recently, I am interested in the geometry which relates to reductive or semi-simple groups in contrast to nilpotent or solvable groups. In particular, I study non-abelian Hodge theory, variations of Hodge structures, lattices in semi-simple Lie groups and locally homogeneous spaces.



Department of Mathematics

Soichiro KATAYAMA

Nonlinear Partial Differential Equations

My research interest is in nonlinear partial differential equations. To be more specific, I am working on the initial value problem for nonlinear wave equations (in a narrow sense), and also for partial differential equations describing the nonlinear wave propagation in a wider sense, such as Klein-Gordon equations and Schroedinger equations.

The initial value problem is to find a solution to a given partial differential equation with a given state at the initial time (a given initial value). However, in general, it is almost impossible to give explicit expression of solutions to nonlinear equations. Therefore, in the mathematical theory, it is important to investigate the existence of solutions and also their behavior when they exist.

If we consider the initial value problem for the equations mentioned above and if the initial value is sufficiently small, the existence of solutions up to arbitrary time (the existence of global solutions) is mainly determined by the power of the nonlinearity. Especially, when the nonlinearity has the critical power, the existence and non-existence of global solutions depend also on the detailed structure of the nonlinear terms. I am interested in this kind of critical case, and studying sufficient conditions for the existence of global solutions and their asymptotic behavior.

Department of Mathematics

Kazunori KIKUCHI

Differential Topology

I have been studying topology of smooth four-dimensional manifolds, in particular interested in homology genera, representations of diffeomorphism groups to intersection forms, and branched coverings. Let me give a simple explanation of what interests me the most, or homology genera. The homology genus of a smooth four-dimensional manifold M is a map associating to each two-dimensional integral homology class [x] of M the minimal genus g of smooth surfaces in M that represent [x]. For simplicity, reducing the dimensions of M and [x] to the halves of them respectively, consider as a two-dimensional manifold the surface of a doughnut, or torus T, and a one-dimensional integral homology class [y] of T. Draw a meridian and a longitude on T as on the terrestrial sphere, and let [m] and [l] denote the homology classes of T represented by the meridian and the longitude respectively. It turns out that [y] = a[m] + b[l] for some integers a and b, and that [y] is represented by a circle immersed on T with only double points. Naturally interesting then is the following question: what is the minimal number n of the double points of such immersions representing [y]? Easy experiments would tell you that, for example, n = 0 when (a,b) = (1,0) or (0,1) and n = 1 when (a,b) = (2,0) or (0,2). In fact, it is proved with topological methods that n = d−1, where d is the greatest common divisor of a and b. It is the minimal number n for T and [y] that corresponds to the minimal genus g for M and [x]. The study on the minimal genus g does not seem to proceed with only topological methods; it sometimes requires methods from differential geometry, in particular methods with gauge theory from physics; though more difficult, it is more interesting to me. I have been tackling the problem on the minimal genus g with such a topological way of thinking as to see things as if they were visible even though invisible.

Department
of
Mathematics

Kohei KIKUTA

Algebraic Geometry, Group Theory

I'm interested in autoequivalence groups of derived categories of algebraic varieties, especially K3 surfaces. The objects of derived categories are complexes of coherent sheaves on algebraic varieties, and the morphisms are given by using morphisms between complexes. Derived categories contain much information of algebraic varieties. Auto-equivalence groups are just like auto-morphism groups of algebraic varieties. The pull-back of automorphisms gives the natural embedding of auto-morphism groups into auto-equivalence groups. Autoequivalence groups of derived categories are interesting objects in also group-theory.

To understand autoequivalence groups, it is not enough to study only the action on the cohomology groups. Then, via homological mirror symmetry originating the string theory in physics, we compare autoequivalence groups and (symplectic) mapping class groups of symplectic manifolds. Rich research history of mapping class groups gives rich ideas to study autoequivalence groups. Actually, motivating the analogue with mapping class groups, I have been studying autoequivalence groups from group-theoretic/ metric geometric/ dynamical viewpoints. But there's still a lot to work on this topic. My main project is to study autoequivalence groups of K3 surfaces by any method in mathematics.

Department
of
Mathematics

Erika KUNO

Topology

My research theme is mainly mapping class groups of surfaces (including non-orientable surfaces) and 3-dimensional handlebodies. In particular, I am interested in exploring them from the viewpoints of geometric group theory. Geometric group theory is a new field among a lot of areas of mathematics and it is progressing significantly. One of the most important problems in geometric group theory is classifying finitely generated groups by “quasi-isometries”. Two finitely generated groups are quasi-isometric if roughly speaking, their word metrics are the same up to linear functions. An interesting part of the geometric group theory is that the properties of the infinite groups are revealed one by one by measuring with a coarse scale of quasi-isometries, but not isometries. Currently, groups which are quasi-isometric to mapping class groups have hardly been found. Then what I am wondering is the question "Which groups are quasi-isometric to the mapping class group?". Based on this big theme, I would like to elucidate properties of mapping class groups, and deepen their understanding.

Department of Mathematics

Yoshihiko MATSUMOTO

Differential Geometry, Several Complex Variables

I am studying “asymptotically symmetric spaces” and the geometry of their asymptotic boundaries.

Asymptotically symmetric spaces are those that “look more and more like a model space as one moves away from any position within the space,” where the model is a highly symmetric non-compact space such as the real or complex hyperbolic spaces. The primary interest in this field is in how the analytic properties of such spaces resemble those of the model spaces in a broad sense, and how they differ in the details.

An important feature of asymptotically symmetric spaces is that they have an asymptotic boundary with its own differential geometric structure, and there is an interplay between the interior space and its boundary. While theories of boundary structures, such as conformal and CR (Cauchy–Riemann) structures, are profound in themselves, they become more interesting when viewed in relation to asymptotically symmetric spaces.

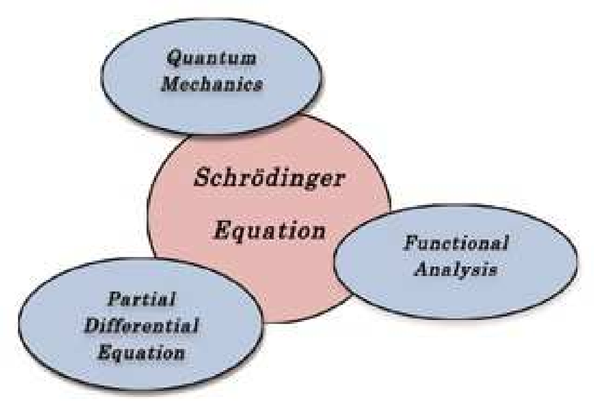
The case that is most extensively studied so far is that of asymptotically real hyperbolic spaces, partly due to interest from physics. However, there are still many unsettled issues even for them, and when we consider the general asymptotically symmetric spaces, it can probably be said that we are facing a largely unexplored wilderness. I’m trying to deepen our understanding by moving back and forth between concrete case studies and abstract considerations.

Department of Mathematics

Haruya MIZUTANI

Partial Differential Equations

The Schrödinger equation is the fundamental equation of physics for describing quantum mechanical behavior. I am working on the mathematical theory of the Schrödinger equation and my research interest includes scattering theory, semiclassical analysis, spectral theory, geometric microlocal analysis and so on. My current research has focused on various estimates such as decay or Strichartz inequalities, which describe dispersive or smoothing properties of solutions and are fundamental for studying linear and nonlinear dispersive equations. In particular, I am interested in understanding quantitatively the influence of the geometry of associated classical mechanics on the behavior of quantum mechanics, via such inequalities.



Department of Mathematics

Tomonori MORIYAMA

Number Theory

I am interested in automorphic forms of several variables. A classical automorphic (modular) form of one variable is a holomorphic function on the upper half plane having certain symmetry. Such functions appear in various branches of mathematics, say notably number theory, and have been investigated by many mathematicians.

There is a family of manifolds called Riemannian symmetric spaces, which is a higher-dimensional generalization of the upper half plane. The set of isometries of a Riemannian symmetric space forms a Lie group G. Roughly speaking, an automorphic form of several variables is a function on a Riemannian symmetric space satisfying the relative invariance under an “arithmetic” subgroup of G and certain differential equations arising from the Lie group G. Studies on automorphic forms of several variables started from C. L. Siegel's works in 1930s and have been developed through interaction with mathematics of the day.

Currently I am working on two themes: (i) the zeta functions attached to automorphic forms and (ii) explicit constructions of automorphic forms, by employing representation theory of reductive groups over local fields. One of the joy in studying this area is to discover a surprisingly simple structure among seemingly complicated objects.

Department of Mathematics

Hiroaki NAKAMURA

Number Theory

Theory of equations has a long history of thousands of years in mathematics, and, passing publication of the famous Cardano-Ferrari formulas in Italian Renaissance, Galois theory in the 19th century established a necessary and sufficient condition for an algebraic equation to have a root solution in terms of its Galois group. My research interest is a modern version of Galois theory, especially its arithmetic aspects. In the last century, the notion of Galois group was generalized to "arithmetic fundamental group" by Grothendieck, and Belyi's discovery (of an intimate relationship between Galois groups of algebraic numbers and fundamental groups of topological loops on hyperbolic curves) undertook a new area of "anabelian geometry". Here are important problems of controlling a series of covers of algebraic curves and their moduli spaces, and Ihara's theory found deep arithmetic phenomena therein. Related also to Diophantus questions on rational points, fields of definitions and the inverse Galois problem, nowadays, there frequently occur important developments as well as new unsolved problems. I investigate these topics, and hope to find new perspectives for deeper understanding of the circle of ideas.

Department of Mathematics

Shohei NAKAMURA

Harmonic Analysis, Real Analysis

I am currently working in Harmonic analysis on Euclidean spaces and real analysis. In particular, my research purpose is to understand the "oscillation" phenomena quantitatively via the inequalities. Let us think about a sequence a_n = (-1)^n for instance. If one sums up the sequence up to n=1000, then the result is 1 as +1 and -1 would be cancelled out. However, if one ignores the cancellation (oscillation) and sums up the sequence |a_n|, then the result is 1000 which seems to have a big difference than before. Although this is the simplest example of oscillation phenomena, the problem would be much more involved once one has the oscillation in more complicated way. For example, one has the oscillation e^{-ixy}, which is the linear and so relatively simpler oscillation, in the definition of the Fourier transform. However, in the definition of the Schrodinger propagator, one has e^{-i (xy + t y^2)}, which is the quadratic oscillation. Hence its treatment would be no longer trivial and one needs to invent a clever way to exploit the complicated oscillation.

Such oscillation phenomena appear in several mathematical fields. For instance, I have already mentioned before, in the theory of partial differential equations as well as in Fourier analysis. Not only in analysis, but one can also find such phenomena in number theory; a problem to count a number of integer solutions of certain Diophantine equation can be boiled down to the question how to capture the "oscillation". Among them, I am in particular interested in how to capture the oscillation of the Fourier transform on a hypersurface. Related to this problem, I am also interested in geometric inequalities with the sharp constant, like Young's convolution inequality.

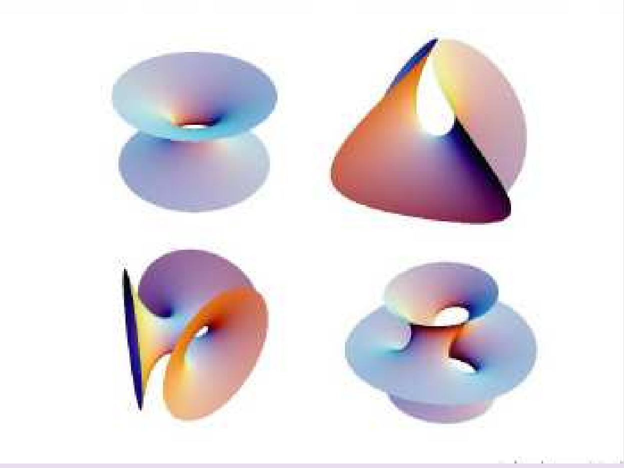
Department of Mathematics

Takahiro OBA

Topology

My research interests lie in the topology of manifolds. In particular, I am interested in that of contact and symplectic manifolds. One of the main problems in this area is how much these geometric structures impose constraints on the topology of those manifolds. One can address this problem by various methods. Among them, I have mainly used fibration-like structures: Lefschetz fibrations and open book decompositions. One can extract information about the topology of contact and symplectic manifolds from fibration-like structures, and vice versa.

I have studied low-dimensional, namely 3-/4-dimensional contact and symplectic manifolds. I have recently started trying to understand higher dimensions. Little is known about this case, because of lack of knowledge on mapping class groups of higher-dimensional manifolds, so this is challenging. With various techniques such as holomorphic curve techniques, now I am diving into the higher-dimensional world.



Department of Mathematics

Hiroyuki OGAWA

Algebraic Number Theory

I have an interest in periodic objects. Expanding rational numbers into decimal numbers is delightful. The decimal number expansion becomes the repeat of a sequence of some integers. I have an appetite for continued fraction expansions, never get tired to calculate it, and want to find continued fractions with sufficiently long period. It is on the way to Gauss' class number one conjecture. Recently, I am studying iteration of rational functions. For a rational function g(x) with rational coefficients, a complex number z with g(g(...g(z)...))=z is called a periodic point on g(x) and is an algebraic number. I expect that number theoretical properties which such an algebraic number z has is described by the rational function g(x). This does not seem to work out anytime, but one can find many rational functions g(x) that describe the Galois group, the class number, the class group, and so on of a periodic point of g(x). I think that this should be surely useful, and calculate like these every day.

Department of Mathematics

Koji OHNO

Algebraic Geometry

When I was a student, I thought I knew number theory, geometry, but algebraic geometry was unfamiliar for me. One may say algebra and geometry are different fields, but you know the theory of quadratic curves and are aware of efficiency of algebraic methods for solving geometric problems. The field called algebraic geometry lies on such a line. When I was studying the theory of quadratic curves, I wondered, "Why do they only treat special equations like quadratics? There are many other equations. But how can they be treated?". When I discovered the answer might lie on this field, I decided to enter this field. The easiest non-trivial equation has the form such as "the second power of y = an equation of x of degree three", which defines the so called "an elliptic curve". The theory of elliptic curve was one of the greatest achievements of nineteenth century and keeps developing today. Recently, the famous Fermat's conjecture has been solved using this theory. The theory of quadratic and elliptic curves involve only two variables x, y. It is natural to think of the equations with many variables. In fact the algebraic geometers are expanding the theory, curves to surfaces and higher dimensional cases these days. Two dimensional version of elliptic curves are called K3 surfaces, which can be treated only using the theory of linear algebra(!) thanks to the Torelli's theorem. These days, the 3-dimensional versions, which is called Calabi-Yau threefolds are fascinating for algebraic geometers like me. Somehow theoretical physicists are also interested in this field. To study Calabi-Yaus by specializing these to ones with a fiber structure (on which field, I'm now working) might be one method, but I have been thinking that a new theory is needed. These days, many intriguing new theories have appeared and one may find more!

Department
of
Mathematics

Shin-ichi OHTA

Differential Geometry, Geometric Analysis

My research subject is geometry, especially differential geometry and geometric analysis related to analysis and probability theory. A keyword of my research is “curvature” which represents how the space is curved. As seen in the difference between the sums of interior angles of triangles in a plane and a sphere, the behavior of the curvature influences various properties of the space (the shapes of triangles, the volume growth of concentric balls, how heat propagates, the behavior of entropy, etc.). This powerful and versatile conception has been applied to Riemannian manifolds, metric spaces, Finsler manifolds, Banach spaces, as well as discrete objects such as graphs.

Department
of
Mathematics

Mamoru OKAMOTO

Nonlinear Partial Differential Equations

My research interest is in nonlinear partial differential equations. Especially, I am working on the Cauchy problem for nonlinear dispersive equations, such as nonlinear Schrödinger and nonlinear wave equations. The Cauchy problem is a problem to find a solution to a differential equation satisfying what are known as initial conditions (initial data) at a certain (initial) time.

For nonlinear problems, it is usually not possible to obtain explicit solutions, so we need to show an existence result of a solution. It then may happen that the solution changes significantly when the initial data change small. However, this phenomenon causes some difficulties on analysis of physical models and differential equations. Hence, it is often required the well-posedness of the Cauchy problem, which ensures that the solution continuously depends on initial data. One of the distinguishing features of linear dispersive equations is that the speed of the spread of solutions depends on its frequency. Therefore, some kind of smoothing effect can be obtained by incorporating the oscillation caused by dispersion due to time evolution into the analysis. Furthermore, by grasping what kind of oscillation is generated from each nonlinear term, we can use the smoothing effect in the analysis of nonlinear dispersive equations.

I am studying the well-posedness of the Cauchy problem for dispersive equations by using the smoothing effect due to such a dispersive nature. Recently, I am also interested in asymptotic behavior of solutions to the Cauchy problem and stochastic dispersive equations.

Department of Mathematics

Shinnosuke OKAWA

Algebraic Geometry

Geometric objects which are described as the collection of solutions of algebraic equation(s), such as ellipses, parabolas, and hyperbolas, are called algebraic varieties. These are the subject of study in the field of algebraic geometry, and I have been working on various problems in this area.

In the early days, I was studying topics about Geometric Invariant theory (GIT) and birational geometry. GIT is a theory about quotients of algebraic varieties by algebraic group actions, and in birational geometry certain "transformation (modification)" of algebraic varieties is studied. The definition of a GIT quotient depends on a choice of a parameter, which is called a stability condition, and one obtains different quotients by changing the stability conditions. Typically these quotients are all birational to each other, and in good situations it turns out that the birational geometry of the quotient variety is complete described in this way.

I have proved several properties of this class of varieties. Recently I am mainly investigating algebraic varieties from categorical points of view. One of my interests is the "irreducible decomposition" of the derived category of coherent sheaves. This in fact is motivated by birational geometry, and important techniques of birational geometry, e.g. the canonical bundle formula, are used. I am also studying non-commutative deformations of algebraic varieties and their moduli spaces. Derived categories again plays a central role, but other interesting topics such as GIT, geometry of elliptic curves, and birational transformation of non-commutative algebraic varieties also show up. Through the study, I found an interesting and unexpected relationship with an old invariant theory which goes back to the end of the 19th century. I am also trying to understand to what extent the derived category of coherent sheaves keeps the geometric information of the original variety.

Department of Mathematics

Kazuto OTA

Number Theory

My research area is number theory. More precisely, I am interested in mysterious relations between L-functions and arithmetic invariants which are attached to algebraic varieties defined over the rationals. Roughly speaking, L-functions are regarded as a vast generalization of the Riemann zeta function, and arithmetic invariants here are rational solutions of algebraic equations, Selmer groups etc. Although there are conjectural formula for general algebraic varieties of several forms nowadays, almost all of them are still unsolved. For example, the BSD conjecture is such a conjecture for elliptic curves (algebraic varieties of dimension one), and it is one of The Millennium Prize Problems by the Clay Mathematics Institute. By this example, we may see difficulty and importance of such conjectures connecting L-functions and arithmetic invariants. I would like to contribute to them. So far, I have studied relations between L-functions and Selmer groups of elliptic curves and elliptic modular forms by p-adic methods, where p is a rational prime number. I will next study to understand higher-dimensional objects as well.

Department of Mathematics

Toru SERA

Ergodic Theory, Probability Theory, Dynamical Systems

My research interest is in ergodic transformations with infinite invariant measures. Typical examples are intermittent maps, i.e., non-uniformly expanding maps with indifferent fixed points. Intermittent maps have been studied as toy models of intermittent phenomena and are important research subjects in mathematics and statistical physics.

In the case of probability-preserving ergodic transformations, Birkhoff's ergodic theorem states that the long-time average of an appropriate observable converges to the space average. In the case of infinite ergodic transformations, Birkhoff's ergodic theorem is not very useful. Nevertheless, many researchers have studied various probabilistic limit theorems. For example, given an appropriate initial distribution for an intermittent map, the long-time average of a suitable observable converges to a generalized arcsine distribution.

My research aim is to deepen and develop probabilistic limit theorems for intermittent maps. For this purpose, I have been focusing on the probabilistic similarities between one-dimensional diffusion processes and intermittent maps. Although they are different from each other in origin, we can use the analysis methods of the former to study the latter under some modifications.

Department of Mathematics

Atsushi TAKAHASHI

Complex Geometry, Algebra, Mathematical Physics

Mathematics and theoretical physics have developed in tandem, mutually inspiring each other. Among them, mirror symmetry, originating from string theory, has had a profound impact on mathematics. The physics of mirror symmetry has been a source of highly intriguing ideas, connecting fields that have been studied individually such as group theory, representation theory, modular forms, number theory, algebraic geometry, symplectic geometry, and more. It has anticipated phenomena unimaginable within mathematics and provided clues to solve important problems.

What particularly interests me is the mathematics of mirror symmetry. I aim to incorporate physical ideas into mathematics, deepening existing mathematics while pioneering new ones. Specifically, my main research goal is to understand the “something” that governs the equivalence of categories of three types of derived categories constructed algebraically, geometrically, and representation-theoretically (homological mirror symmetry), as well as isomorphisms of flat structures (classical mirror symmetry). Additionally, applying the results and insights gained here to algebraic geometry, particularly qualitative classification of “algebraic spaces” and deriving quantitative invariants, is also one of my objectives.

Recently, under the notion of mirror symmetry where “algebraic spaces” equals “derived categories + Bridgeland stability,” I have begun to explore the idea of identifying “data” with “spaces,” with the belief that concepts and results developed in mirror symmetry research can be applied to data science, thus embarking on applied research as well.

Department
of
Mathematics

Shuichiro TAKEDA

Number Theory, Representation Theory

My research area is called the Langlands program. The Langlands program is at the crossroad of several important branches of modern mathematics such as number theory, representation theory and harmonic analysis. Because of this, it is considered as one of the most difficult areas of mathematics, but at the same time it has been producing various interesting research outcomes.

The major object of study in the Langlands program is called automorphic representations and their L-functions. Automorphic representations are representations of certain matrix groups called reductive groups, and considered as a vast generalization of classical modular forms. An automorphic representation is further decomposed into local representations such as representations of p-adic groups. I have been studying these representations by using the method called theta correspondence, which allows one to construct a representation of one reductive group out of a representation of another. Via this method, I have obtained numerous interesting results in the theory of automorphic representations.

Department
of
Mathematics

Naohito TOMITA

Real Analysis

My research field is Fourier analysis, and I am particularly interested in the theory of function spaces. Fourier series were introduced by J. Fourier(1768-1830) for the purpose of solving the heat equation. Fourier considered as follows: "Trigonometric series can represent arbitrary periodic functions". However, in general, this is not true. Then, we have the following problem: "When can we write a periodic function as an infinite (or finite) sum of sine and cosine functions?". Lebesgue space which is one of function spaces plays an important role in this classical problem. Here Lebesgue space consists of functions whose p-th powers are integrable. In this way, function spaces are useful for various mathematical problems. As another example, modulation spaces were recently applied to pseudodifferential operators which are important tool for partial differential equations, and my purpose is to clarify their relation.

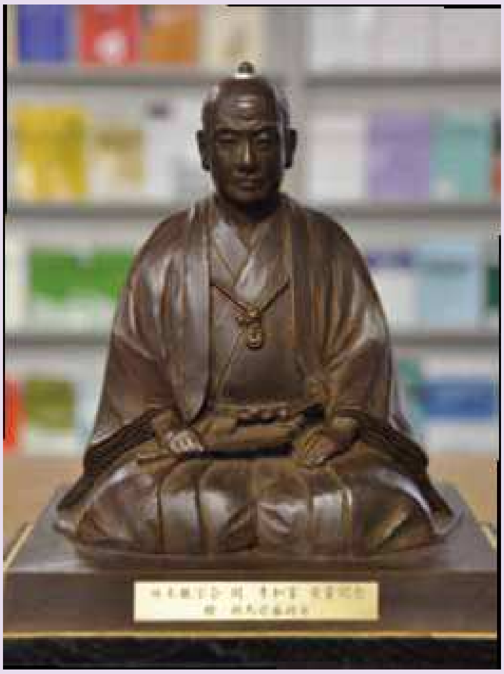
$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Department
of
Mathematics

Motoo UCHIDA

Algebraic Analysis, Microlocal Analysis

My research field is algebraic analysis and micro-local analysis of partial differential equations. The view point of micro-local analysis (with cohomology) is a new important point of view in analysis introduced by Mikio Sato in the early 1970s. Thinking from a micro-local point of view helps us to well understand a number of mathematical phenomena (at least for PDE) and to find a simple hidden principle behind them. Even for some classical facts (scattered as well known results) we can sometimes find a new unified way of understanding from a micro-local or algebro-analytic viewpoint.



Department
of
Mathematics

Takao WATANABE

Algebraic Number Theory

My current interest is the Geometry of Numbers. The Geometry of Numbers was founded by Hermann Minkowski in the beginning of the 20th century. Minkowski proved a famous theorem known as "Minkowski's convex body theorem", which asserts that "there exists a non-zero integer point in V if V is an o-symmetric convex body in the n-dimensional Euclidean space whose volume is greater than 2^n". When V is an ellipsoid, this theorem is refined as follows. Let A be a non-singular 3 by 3 real matrix and K(c) the ellipsoid consisting of points x such that the inner product (Ax, Ax) is less than or equal to c > 0. For i = 1,2,3, we define the constant c_i as the minimum of c > 0 such that K(c) contains i linearly independent integer points. Then c_1, c_2, c_3 satisfies the inequality c_1c_2c_3 <= 2|det A|^2. This is called "Minkowski's second theorem". A similar inequality holds for any n-dimensional ellipsoid. Namely, if A is a non-singular n by n real matrix and K(c) is the n-dimensional ellipsoid defined by (Ax, Ax) <= c, we can define c_i for i = 1,2, ..., n as the minimum of c > 0 such that K(c) contains i linearly independent integer points. Then the inequality c_1c_2...c_n <= h(n)|det A|^2 holds for any A. The optimal upper bound h(n) does not depend on A, and is called Hermite's constant. We know h(2) = 4/3, h(3) = 2, h(4)= 4, ..., h(8) = 256, but h(n) for a general n is not known. A recent major topic of this research area is the determination of h(24). In 2003, Henry Cohn and Abhinav Kumar proved that h(24) = 4^24. (Incidentally, h(3) was essentially determined by Gauss in 1831, and h(8) was determined by Blichfeldt in 1953. If you would determine h(9), then your name would be recorded in treatises on the Geometry of Numbers.) Now I study (an analogue of) the Geometry of Numbers on algebraic homogeneous spaces. One of my results is a generalization of Minkowski's second theorem to a Severi-Berauer variety. In addition, I am interested in the reduction theory of arithmetic subgroups, automorphic forms, the algebraic theory of quadratic forms and Diophantine approximation.

Department of Mathematics

Katsutoshi YAMANOI

Complex Analysis, Complex Geometry

My research interest is Complex geometry and Complex analysis, both from the view point of Nevanlinna theory. In the geometric side, I am interested in the conjectural second main theorem in the higher dimensional Nevanlinna theory for entire holomorphic curves into projective manifolds. Also I am interested in the behavior of Kobayashi pseudo-distance of projective manifolds of general type. These problems are related to an algebraic geometric problem of bounding the canonical degree of algebraic curves in projective manifolds of general type by the geometric genus of the curves. In the analytic side, I am interested in classical problems of value distribution theory for meromorphic functions in the complex plane.

Department of Mathematics

Kouji YANO

Probability Theory, Ergodic Theory

Probability theory contains several limit theorems such as the law of large numbers, the law of iterated logarithm, the central limit theorem, the Poisson law of small numbers, and the arcsine law. They are mathematical theories about limit properties in long time for stochastic processes which change randomly as time passes.

I have studied extensions of the arcsine law to one-dimensional diffusion processes and to random dynamical systems with fixed points which are indifferent-in-mean. The arcsine law describes the rate where a stochastic process is attracted to positive and negative infinity, and it also describes the rate where a dynamical system with two indifferent fixed points is attracted to them. It is an interesting problem where the theory of stochastic processes and the infinite ergodic theory intersect.

I have also studied the penalisation problem, a limit theorem where the stochastic process considered is conditioned to prevent from a certain behavior for a long time. Compared to the original stochastic process, the limit process is absolutely continuous in finite time but is singular in infinite time, and the Radon-Nikodym density is given by a martingale converging to zero. The way of taking the limit here is called a clock. If we adopt instead of a constant clock a random clock such as an exponential clock and a hitting time clock, we encounter various martingales. Looking at a stochastic process from different viewpoints, we came to find its interesting structures.

Department of Mathematics

Takehiko YASUDA

Algebraic Geometry, Singularity Theory

My main research object is singularities of algebraic varieties. An algebraic variety is a "figure" formed by solutions of algebraic equations. Such a figure often has points where the figure is sharp-pointed or intersects itself. Singularities make the study of an algebraic variety difficult. However since they often appear under various constructions, it is important to study them. Also singularities are interesting research object themselves. More specifically, I am interested in resolution of singularities, the birational-geometric aspect of singularities, the McKay correspondence.

Although these are classical research areas, changing a viewpoint or the setting of a problem, one can sometimes find new phenomenon. Such a discovery is the greatest pleasure in my mathematical research. To pursue research, I use various tools like motivic integration, Frobenius maps, moduli-theoretic blowups, non-commutative rings, and sometimes make ones by myself. Recently I am fascinated by mysterious behaviors of singularities in positive characteristic (a world where summing up several 1's gives 0.)

Department of Mathematics

Masahiko YOSHINAGA

Algebra, Topology, Combinatorics

The central object of my research is the hyperplane arrangement. A hyperplane arrangement is a collection of (n-1)-dimensional subspaces in a n-dimensional space. Such objects appear many area of mathematics. Recently, I am mainly working in the following topics.

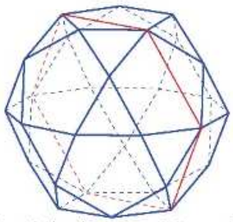
(1) The freeness of the module of logarithmic vector fields and the construction of the basis.

(2) Topology of the Milnor fiber and covering spaces.

(3) Lattice points counting and the characteristic quasi-polynomials.

(1) has connections with algebraic geometry and representation theory, and recently, is influenced by the study of quantum integrable systems. In (2), the icosidodecahedral arrangement (Figure) plays crucial role in recent studies. (3) has very rich connections with many other topics such as the theory of polytopes, arrangements of tori, and generalizations of the notion of Tutte polynomial for graphs and matroids.

I am also interested in the notion of the magnitude of metric spaces introduced by categorists, and more generally, categorification of combinatorial phenomena. My interests also include the notion of "periods" that are real numbers with integral expressions.



Icosidodecahedron and diagonals

SEMINARS and COLLOQUIA

●ALGEBRA

Department
of
Mathematics

Number Theory Seminar

Number theory seminar at Osaka University is a seminar for faculty members and graduate students of Osaka University or researchers studying nearby Osaka University. The subject of the seminar covers wide topics concerning Number theory, especially, algebraic number theory and analytic number theory, modular forms, arithmetic geometry, representation theory and algebraic combinatorics. In this seminar, we have reports of new results on these topics and we exchange ideas and technics of our research.

Department
of
Mathematics

Algebraic Geometry Seminar

The seminar is held two or three times a month and each time one speaker gives a talk of 90 minutes. After a talk, we have time for questions and discussion. The purpose of the seminar is to learn important results by active researchers in Algebraic Geometry and related fields, providing new perspectives on the areas through lectures and discussions. We also have survey lectures by experts for graduate students and young researchers. We have guest speakers not only from domestic universities but also from foreign countries, reflecting various aspects of the research area.

●GEOMETRY

Department
of
Mathematics

Geometry Seminar

This seminar on Mondays is intended for talks that will be of interest to a wide range of geometers. Topics discussed include Riemannian, complex, and symplectic geometry; PDEs on manifolds; mathematical physics.

Department
of
Mathematics

Topology Seminar

This seminar focuses on various aspects of low-dimensional topology. Our major topics are 2-, 3-, and 4-dimensional manifold theory, knot theory, geometric group theory, hyperbolic geometry, Riemann surfaces, and transformation groups. In addition to our group members from the math department, associated faculty members from other departments and their graduate students are also regular participants.

●ANALYSIS

Department of Mathematics

Seminar of Differential Equations

Our seminar is held on every Friday from 15:30 to 17:00. One of the features of the seminar is to cover a wide variety of topics on Qualitative Analysis of Differential Equations. In fact, we are interested in ordinary differential equations, partial differential equations, linear differential equations, nonlinear differential equations and so on. Lecturers are invited from not only domestic universities but also foreign countries and present us their original results or survey of recent development of their fields. Furthermore, this seminar provides opportunities to give a talk for our colleagues and Ph.D. students majoring in differential equations. Moreover, we should mention that we are pleased to have participants from other universities located closed to ours. In this way, we communicate with each other and try to contribute to the progress of the theory of differential equations.

Department of Mathematics

Dynamics and Fractals Seminar

Researchers and students working on various fields related to dynamical systems and fractals attend this seminar. We meet once a month for approximately 90 minutes. Each talk on his/her research is followed by discussions among all participants.

Department of Mathematics

Seminar on Probability

The probability theory group in the Graduate School of Science and the Graduate School of Engineering Science organizes the Seminar on Probability. The topics of this seminar are the following:

1. Probability theory, stochastic analysis, infinite dimensional analysis, and related problems arising from other areas of mathematics such as real analysis, differential equations, and differential geometry.
2. Problems related to probability theory in ergodic theory, dynamical systems, stochastic control, mathematical finance, etc.

We welcome visits from many researchers from other universities, both domestic and international.

●COLLOQUIA

Department of Mathematics

Mathematics Colloquium

Colloquia take place on Monday afternoon at 17:00 in Room E404. They are directed toward a general mathematical audience. In particular, one of the functions of these Colloquia is to inform non-specialists and graduate students about recent trends, ideas and results in some area of mathematics, or closely related fields.