Summary

Mathematics, originally centered in the concepts of number, magnitude, and form, has long been growing since ancient Egyptian times to the 21st century. Through the use of abstraction and logical reasoning, it became an indispensable tool not only in natural sciences, but also in engineering and social sciences. Recently, the remarkable development of computers is now making an epoch in the history of mathematics.

Department of Mathematics is one of the six departments of Graduate School of Science, Osaka University. It consists of 6 research groups, all of which are actively involved in the latest developments of mathematics. Our mathematics department has ranked among the top seven in the country.

The department offers a program with 32 new students enrolled annually leading to post-graduate degrees of Masters of Science. The department also offers a Ph.D. program with possibly 16 new students enrolled annually. In addition, there is a special program that excellent undergraduate students with strong mathematical background can be enrolled in the graduate school before graduation. So far, ten students successfully used this program.

Graduate courses are prepared so as to meet various demands of students. Besides introductory courses for first year students, a number of topics courses are given for advanced students. Students learn more specialized topics from seminars under the guidance of thesis advisors.

Our department has our own library equipped with about 500 academic journals and 50,000 books in mathematics, both of which graduate students can use freely. Also by an online system, students as well as faculty members can look up references through Internet.

Research Groups


Areas of Research


Faculty Members:

Professors (16)
Shin-ichi DOI, Akio FUJIWARA, Ryushi GOTO, Nakao HAYASHI, Osamu KOBAYASHI, Norihito KOISO, Kazuhiro KONNO, Takehiko MORITA, Hiroaki NAKAMURA, Tatsu NISHITANI, Keiji OGUSIO, Ken’ichi OHISHIKA, Hiroshi SUGITA, Atsushi TAKAHASHI, Takao WATANABE, Katsutoshi YAMANOI.

Associate Professors (14)
Ichiro ENOKI, Masaaki FUKASAWA, Masashi ISHIDA, Eiko KIN, Hideki MIYACHI, Tomonori MORIYAMA, Tadashi OCHIAI, Hiroki SUMI, Hideaki SUNAGAWA, Joe SUZUKI, Naohito TOMITA, Motoo UCHIDA, Seidai YASUDA, Takehiko YASUDA.

Assistant Professors (6)
Yasuhiro HARA, Takao IOHARA, Shinichiroh MATSUO, Haruya MIZUTANI, Hiroyuki OGAWA, Koji OHNO.

Cooperative Members in Osaka University

Professors (7)
Susumu ARIKI, Takayuki HIBI, Katsuhisa MIMACHI, Kenji NAKANISHI, Shinji ODANAKA, Katsuhiko UNO, Masaaki WADA.

Associate Professors (6)
Tsuyoshi CHAWANYA, Daisuke FURIHATA, Kei MIKI, Satoshi MURAI, Kiyokazu NAGATOMO, Yosuke OHYAMA.
Shin-ichi DOI

Partial Differential Equations

Partial differential equations have their origins in various fields such as mathematical physics, differential geometry, and technology. Among them I am particularly interested in the partial differential equations that describe wave propagation phenomena: hyperbolic equations and dispersive equations. A typical example of the former is the wave equation, and that of the latter is the Schroedinger equation. For many years I have studied basic problems for these equations: existence and uniqueness of solutions, structure of singularities of solutions, asymptotic behavior of solutions, and spectral properties. Recently I make efforts to understand how the singularities of solutions for Schroedinger equations or, more generally, dispersive equations propagate. The center of this problem is to determine when and how the singularities of solutions for the dispersive equations can be described by the asymptotic behavior of solutions for the associated canonical equations.

Ichiro ENOKI

Complex Differential Geometry

A complex manifold is, locally, the world build out of open subsets of complex Euclid spaces and holomorphic functions on them. If two holomorphic functions are defined on a connected set and coincide on an open subset, then they coincide on the whole. Complex manifolds inherit this kind of property from holomorphic functions. That is, they are stiff and hard in a sense. It seems to me that complex manifolds are not metallically hard but have common warm feeling with wood or bamboo, which have grain and gnarl. Analytic continuations, as you learned in the course on the function theory of complex variables, is analogous to the process of growth of plants. Instead of considering whole holomorphic functions, a class of complex manifold can be build out of polynomials. This is the world of complex algebraic manifolds, the most fertile area in the world of complex manifolds. To complex algebraic manifolds, since they are algebraically defined, algebraic methods are of course useful to study them. In certain cases, however, transcendental methods (the word "transcendental" means only "not algebraic") are powerful. For example, one of the simplest proof for the fundamental theorem of algebra is given by the function theory of one complex variable. These two methods have been competing with each other since the very beginning of the history of the study of complex manifolds. This competing seems to me the prime mover of the development of the theory of complex manifolds. Comparing the world of complex manifold to the earth, the world of complex algebraic manifold is to compare to continents, and the boundary to continental shells. The reason I wanted to begin to study complex manifolds was I heard the Kodaira embedding theorem, which characterizes complex algebraic manifolds in the whole complex manifolds. The place I begin to study is, however, something like the North Pole or the Mariana Trench. Now the center of my interest is in the study of complex algebraic manifolds by transcendental methods. (Thus I have reached land but I found this was a jungle.)
I am mainly interested in asymptotic distributions of stochastic processes. It is often that we encounter an equation describing a natural or social phenomenon which is so complicated that we can derive essentially nothing from its precise form. By taking a limit in an appropriate way, we are however, sometimes able to get a simple asymptotic solution which is enough to solve practical problems underlying the equation. For example, we do not need to solve the Schroedinger equation in order to calculate ballistics, even though the quantum mechanics is behind all the dynamics. This is because by letting Planck's constant, which is very small, converge to 0, we actually come back to the classical world of the Newton mechanics.
Ryushi GOTO

Geometry

My research interest is mostly in complex and differential geometry, which are closely related with algebraic geometry and theoretical physics. My own research started with special geometric structures such as Calabi-Yau, hyperKähler, G2 and Spin(7) structures. These four structures exactly correspond to special holonomy groups which give rise to Ricci-flat Einstein metrics on manifolds. It is intriguing that these moduli spaces are smooth manifolds on which local Torelli type theorem holds. In order to understand these phenomena, I introduce a notion of geometric structures defined by a system of closed differential forms and establish a criterion of unobstructed deformations of structures. When we apply this approach to Calabi-Yau, hyperKähler, G2 and Spin(7) structures, we obtain a unified construction of these moduli spaces. At present I also explore other interesting geometric structures and their moduli spaces.

Yasuhiro HARA

Topology

The field of my study is topology and, especially, I study the theory of transformation groups. The Borsuk-Ulam theorem is one of famous theorems about transformation groups. This theorem is often taken up as an application in elementary lectures about the homology theory. The content of the theorem is as follows: for every continuous map from the n-dimensional sphere to the n-dimensional Euclidian space, there exists a point such that the map takes the same value at the point and at the antipodal point. A famous application of this theorem is the following: “On the earth, there is a point such that the temperature and humidity at the point are the same as those at the antipodal point.” We consider a free action of a group of order two on the n-dimensional sphere to prove the Borsuk-Ulam theorem. Then for any equivariant map (any continuous map which preserves the structure of the group action) from the sphere to itself, the degree of the map is odd. By using this fact, we obtain the Borsuk-Ulam theorem. In the case of the Borsuk-Ulam theorem, we consider spheres and free actions of a group of order two. Actually, when we consider other manifolds and actions of other groups, there are some restrictions of homotopy types of equivariant maps. I study such restrictions of homotopy types of equivariant maps by using the cohomology theory, and I study relationships between homotopy types of equivariant maps and topological invariants.
Nakao HAYASHI

Partial Differential Equations

I am interested in asymptotic behavior in time of solutions to nonlinear dispersive equations (1 D nonlinear Schroedinger, Benjamin-Ono, Korteweg-de Vries, modified Korteweg-de Vries, derivative nonlinear Schroedinger equations) and nonlinear dissipative equations Complex Landau-Ginzburg equations, Korteweg-de Vries equation on a half line, Damped wave equations with a critical nonlinearity). These equations have important physical applications. Exact solutions of the cubic nonlinear Schroedinger equations and Korteweg-de Vries can be obtained by using the inverse scattering method. Our aim is to study asymptotic properties of these nonlinear equations with general setting through the functional analysis. We also study nonlinear Schroedinger equations in general space dimensions with a critical nonlinearity of order 1+2/n and the Hartree equation, which is considered as a critical case and the inverse scattering method does not work. On 1995, Pavel I. Naumkin and I started to study the large time behavior of small solutions of the initial value problem for the non-linear dispersive equations and we obtained asymptotic behavior in time of solutions and existence of modified scattering states to nonlinear Schroedinger with critical and subcritical nonlinearities. It is known that the usual scattering states in L^2 do not exist in these equations. Recently, E.I.Kaikina and I are studying nonlinear dissipative equations (including Korteweg-de Vries) on a half line and some results concerning asymptotic behavior in time of solutions are obtained.

Takao IOHARA

Nonlinear Partial Differential Equations

My research interest is concerned with nonlinear partial differential equations appearing in fluid mechanics. The current research topic is the equations of the motion of viscous incompressible fluid which has free moving surface. The motion of viscous incompressible fluid is governed by the Navier-Stokes equations, which are not easy to solve because of their nonlinearity. The free moving surface adds another nonlinearity to the problem and the study of it needs more elaborate technique than the problems on fixed domain.
Masashi ISHIDA

Geometry

My research interest is in the interaction between topology and differential geometry. The existence and non-existence of Einstein metrics on manifolds is one of the main problems in differential geometry. In dimension four, some topological obstructions to the existence of Einstein metrics are known. However, a topological obstruction to the existence of Einstein metrics in higher dimensions is still unknown. In this sense, Einstein four-manifolds are special and interesting. On the other hand, from the topological point of view, the world of four-manifolds is also special and interesting. For instance, the Seiberg-Witten theory enables us to prove that there is a topological four-manifold admitting infinitely many exotic smooth structures. I am interested in the relation between the existence of Einstein metrics and exotic smooth structures. For example, by using the Seiberg-Witten theory, I constructed examples of topological four-manifolds admitting smooth structures for which Einstein metrics exist, but also admitting smooth structures for which no Einstein metric exists. Such a phenomenon is still unknown in higher dimensions. Moreover, Einstein metrics are closely related to the Ricci flow. The Ricci flow is a nonlinear PDE which deforms metrics by evolving them in the direction of minus the Ricci tensor. I am also interested in the Ricci flow and its generalizations.

Kazunori KIKUCHI

Differential Topology

I have been studying topology of smooth four-dimensional manifolds, in particular interested in homology genera, representations of diffeomorphism groups to intersection forms, and branched coverings. Let me give a simple explanation of what interests me the most, or homology genera. The homology genus of a smooth four-dimensional manifold M is a map associating to each two-dimensional integral homology class [x] of M the minimal genus g of smooth surfaces in M that represent [x]. For simplicity, reducing the dimensions of M and [x] to the halves of them respectively, consider as a two-dimensional manifold the surface of a doughnut, or torus T, and a one-dimensional integral homology class [y] of T. Draw a meridian and a longitude on T as on the terrestrial sphere, and let [m] and [l] denote the homology classes of T represented by the meridian and the longitude respectively. It turns out that [y] = a[m] + b[l] for some integers a and b, and that [y] is represented by a circle immersed on T with only double points. Naturally interesting then is the following question: what is the minimal number n of the double points of such immersions representing [y]? Easy experiments would tell you that, for example, n = 0 when (a,b) = (1,0) or (0,1) and n = 1 when (a,b) = (2,0) or (0,2). In fact, it is proved with topological methods that n = d – 1, where d is the greatest common divisor of a and b. It is the minimal number n for T and [y] that corresponds to the minimal genus g for M and [x]. The study on the minimal genus g does not seem to proceed with only topological methods; it sometimes requires methods from differential geometry, in particular methods with gauge theory from physics; though more difficult, it is more interesting to me. I have been tackling the problem on the minimal genus g with such a topological way of thinking as to see things as if they were visible even though invisible.
Eiko KIN

Geometric Topology

I am interested in the mapping class groups on surfaces. The most common elements in the mapping class groups are so called pseudo-Anosovs. I try to understand which pseudo-Anosovs are the most simplest one. Let me explain my goal more clearly. Pseudo-Anosovs possess many chaotic (and beautiful) properties from the view points of the dynamical systems and hyperbolic geometry. There are some quantities which reflect those complexities of pseudo-Anosovs. (Topological) entropies and (hyperbolic) volumes are examples. We fix a surface (say a closed surface of genus 5 which is my favorite one) and we consider the set of entropies (the set of volumes) coming from the pseudo-Anosov elements on the surface. Then one can see that there exists a minimum of the set. That is, we can talk about the pseudo-Anosovs with the minimal entropies (with the minimal volumes). I would like to know which pseudo-Anosov achieves the minimal entropy/ minimal volume. Recently, Gabai, Meyerhoff and Milley determined hyperbolic 3-manifolds with very small volume. Intriguingly, the result implies that those hyperbolic 3-manifolds are obtained from a single hyperbolic 3-manifold by Dehn filling. Some experts call such a single 3-manifold the "magic manifold". Said differently, the magic manifold is a parent manifold of the hyperbolic 3-manifolds with very small volume. It seems likely that we have the same story in the world of pseudo-Anosovs with the very small entropies. This conjecture is based on recent works of myself and other specialists. We note that there are infinitely many topological types of surfaces. (For example, a family of closed orientable surface of genus g). For the mapping class group of each surface, we know that there exists a pseudo-Anosov element with the minimal entropy. Thus, of course, there exist infinitely many pseudo-Anosov elements with the minimal entropies. It might be true that all minimizers are obtained from the magic 3-manifold. When I work on this project, I sometimes think of our universe.

Osamu KOBAYASHI

Differential Geometry

My research field is differential geometry of smooth manifolds. I prefer intrinsic geometry rather than geometry of submanifolds, though I have some works on maximal surfaces in the Minkowski 3-space. The keyword is curvature. In particular I am interested in the scalar curvature and Ricci curvature. As for the scalar curvature, I introduced the Yamabe invariant in 1980's. Problems relevant to this invariant of smooth manifolds are difficult to solve but most fascinating to me. This is conformal geometric aspect of my research. Meanwhile my study includes differential geometry from projective geometric point of view. To be specific I am working on Ricci curvature of a torsion-free affine connection with a parallel volume element. Connection together with curvature is recognized as the most fundamental concept in differential geometry. In spite of its importance there seem yet very few reseraches on general affine connections.
Norihito KOISO

Differential Geometry

I’m interested in “beautiful” curves and surfaces, and research on their properties and motion. For example, what a shape a piano wire will take when you bend it? what a shape a soap film will take when you bounds it with a wire? These are very classic problem in mathematics, but there are still many interesting open problems. Let’s make a cylinder by a soap film with two circles boundary. If you blow air in the interior of the cylinder, the cylinder will swell out. When the quantity of air is small, we know the shape: it is a rotational surface. However, when the quantity of air is large, we don’t know the shape: it is not rotationally symmetric. We can see the shape by physical or computer experimentation, but we know almost nothing mathematically. Let’s throw a bent piano wire. We know that the piano wire will become straight line finally, but we don’t know the shapes it takes on the way. I’m researching on such a problems as differential geometry with help of analysis.

Kazuhiro KONNO

Complex Algebraic Geometry

Algebraic Geometry is a branch of Mathematics studying, by means of algebraic methods, the geometry of figures defined by simultaneous algebraic equations in several variables. You may say that you are not familiar with Algebraic Geometry. But you already know many beautiful plane curves such as an ellipse, a parabola and a hyperbola; they are in fact our jewels — algebraic varieties. As you learned in high school, various problems on the geometry of plane curves, e.g., how two curves intersect or contact, can be solved by considering simultaneous equations. Studying figures in such a way is nothing but the algebraic geometry. Algebraic equations, however, are not so simple; it is not known so far even how to solve simultaneous quadratic equations, whilst the method for linear equations are well established as you learned in the course of Linear Algebra, and many beautiful algebraic varieties are usually given by quadratic equations. Because in general we cannot draw figures of varieties on the black board, unlike ellipses or parabolas, it requires such and such training in order to be able to touch and feel them. For example, my favorite algebraic surfaces are 4 dimensional objects and, therefore, cannot be realized in our 3 dimensional space. If you are interested in meeting them in reality, the best way is to start and enjoy learning Algebraic Geometry.
I am studying Donaldson theory in 4-manifold topology from the viewpoint of geometric analysis.

Donaldson theory considers the interplay between 4-manifolds and the moduli spaces of anti-self-dual connections on them. Anti-self-dual connections are solutions of the anti-self-dual equations, the partial differential equations of geometric origin. Then, my standpoint is to investigate the moduli spaces by using both geometric studies of 4-dimensional manifolds and analytical techniques of the anti-self-dual equations. I am especially interested in the case when the moduli spaces are of infinite dimension. For example, I studied the anti-self-dual equations on the cylinder without considering any boundary conditions.

From a broader perspective, I have always been fascinated with three things: Infinity, Spaces, and Randomness. What I have been studying is infinite dimensional spaces, which is a hybrid of Infinity and Spaces. Next I want Randomness to go on the stage.

Speaking casually, I love mathematics, because she always says, Let there be light.

My research interests are in Teichmüller theory and geometric structures (especially Hyperbolic geometry) on manifolds, and their applications. I am recently interested in the mathematical phenomena which happen when geometric structures on surfaces (manifolds) are degenerating.

For instance, one of our problems is how the geometrical quantities behave under degenerations. Teichmüller theory is studied and applied in various fields including Complex analysis, Topology, Differential geometry and Algebraic geometry.
Haruya MIZUTANI

Partial differential equations

The Schrödinger equation is the fundamental equation of physics for describing quantum mechanical behavior. I am working on the mathematical theory of the Schrödinger equation and my research interest includes scattering theory, semiclassical analysis, spectral theory, geometric microlocal analysis and so on. My current research has focused on various estimates such as decay or Strichartz inequalities, which describe dispersive or smoothing properties of solutions and are fundamental for studying linear and nonlinear dispersive equations. In particular, I am interested in understanding quantitatively the influence of the geometry of associated classical mechanics on the behavior of quantum mechanics, via such inequalities.

Takehiko MORITA

Ergodic Theory

I specialize in ergodic theory. To be more precise, I am studying statistical behavior of dynamical systems via thermodynamic formalism and its applications.

Ergodic theory is a branch of mathematics that studies dynamical systems with measurable structure and related problems. Its origins can be found in the work of Boltzmann in the 1880s which is concerned with the so called Ergodic Hypothesis. Roughly speaking the hypothesis was introduced in order to guarantee that the system considered is ergodic i.e. the space averages and the long time averages of the physical observables coincide. Unfortunately, it turns out that dynamical systems are not always ergodic in general. Because of such a background, the ergodic problem (= the problem to determine a given dynamical system is ergodic or not) has been one of the important subjects since the theory came into existence. In nowadays ergodic theory has grown to be a huge branch and has applications not only to statistical mechanics, probability, and dynamical systems but also to number theory, differential geometry, functional analysis, and so on.
I am interested in automorphic forms of several variables. A classical automorphic (modular) form of one variable is a holomorphic function on the upper half plane having certain symmetry. Such functions appear in various branches of mathematics, say notably number theory, and have been investigated by many mathematicians.

There is a family of minifolds called Riemannian symmetric spaces, which is a higher-dimensional generalization of the upper half plane. The set of isometries of a Riemannian symmetric space forms a Lie group G. Roughly speaking, an automorphic form of several variables is a function on a Riemannian symmetric space satisfying the relative invariance under an "arithmetic" subgroup of G and certain differential equations arising from the Lie group G. Studies on automorphic forms of several variables started from C. L. Siegel's works in 1930s and have been developed through interaction with mathematics of the day.

Currently I am working on two themes: (i) the zeta functions attached to automorphic forms and (ii) explicit constructions of automorphic forms, by employing representation theory of reductive groups over local fields. One of the joy in studying this area is to discover a surprisingly simple structure among seemingly complicated objects.
Tatsuo NISHITANI

Partial Differential Equations

When you throw a stone into a pond then a ripple on the water propagates in all directions. The propagation is governed by a partial differential equation, the wave equation. Maxwell proposed partial differential equations (Maxwell equations) which the electric field vector and the magnetic field vector satisfy and he deduced that these two fields propagate with the speed of light in vacuum analyzing the equations. Partial differential equations which govern phenomena that a small change occurred in a space fulfilled by some material propagates in all directions with finite speed are called hyperbolic partial differential equations, as the wave equation or Maxwell equations. I am mainly interested in characterizing hyperbolic partial differential equations and in studying properties of solutions to hyperbolic equations. Hyperbolic equations which remain hyperbolic for any choice of lower order terms enjoy a beautiful structure which closely related to the spectrum of the representative of the Hessian (of the principal part) with respect to the standard symplectic form. In these studies, you may wonder, the inequalities are much important than the equalities. I quote a favorite phrase of mine from the Introduction by Peter D. Lax in the collected works (vol II) of Jean Leray (mathematicians who made great contributions to hyperbolic partial differential equations).

"Since a priori estimates lie at the heart of most his arguments, many of Leray's papers contain symphonies of inequalities; sometimes the orchestration is heavy, but the melody is always clearly audible."

Tadashi OCHIAI

Number theory, Arithmetic Geometry

I study Number theory and Arithmetic Geometry. In the research of Number theory, we study not only properties of integers and rational numbers, but every kinds of problems related to integers. For example, we are very much interested in rational points of algebraic varieties defined over the field of rational numbers. We have a long history of our research and we had a great progress on such problems through the proof of Fermat’s conjecture by Wiles and the proof of Mordell’s conjecture by Faltings in 20th century. My subject of research was first the study of geometry of arithmetic algebraic varieties using the l-adic etale cohomology of the variety and the Lefshetz trace formula. More recently, I am interested in the study of the special values of zeta functions via the philosophy of Iwasawa theory. My project is to study Iwasawa theory from the view point of Galois deformations. I think that Number theory is full of surprise as we see a lot of unexpected relations between different kinds of objects.
Hiroyuki OGAWA

Number Theory

I have an interest in periodic objects. Expanding rational numbers into decimal numbers is delightful. The decimal number expansion becomes the repeat of a sequence of some integers. I have an appetite for continued fraction expansions, never get tired to calculate it, and want to find continued fractions with sufficiently long period. It is on the way to Gauss’ class number one conjecture. Recently, I am studying iteration of rational functions. For a rational function $g(x)$ with rational coefficients, a complex number $z$ with $g(g(...g(z)...))=z$ is called a periodic point on $g(x)$ and is an algebraic number. I expect that number theoretical properties which such an algebraic number $z$ has is described by the rational function $g(x)$. This does not seem to work out anytime, but one can find many rational functions $g(x)$ that describe the Galois group, the class number, the class group, and so on of a periodic point of $g(x)$. I think that this should be surely useful, and calculate like these every day.

Keiji OGUISO

Algebraic Geometry

My specialty is algebraic geometry. I am interested in K3 surfaces, elliptic surfaces and their higher dimensional analogues, Calabi-Yau manifolds in wider sense and fiber spaces. These objects play important roles in the classification theory. They also naturally relate with many of mathematics such as lattice theory, number theory, group theory, complex geometry, complex dynamics and so on, via period and symmetry. Such interaction makes study of Calabi-Yau manifolds richer and more exciting, and attracts me very much. We have a complete description of K3 surfaces and elliptic surfaces. However, if one would study them from a fresh right view, then one could find many unexpected, beautiful properties. For instance, mysterious relation between finite symplectic automorphism groups of K3 surfaces and the Mathieu group of degree 23 (Mukai), very impressive construction of K3 surface automorphism with Siegel disk via Salem number (McMullen), surprising role of the theory of Mordell-Weil lattice in sphere packing problem (Shioda) and so on. Though less impressive than above mentioned three results, I was very exciting when I found the fact that K3 surfaces with infinite automorphism group are always dense in any non-trivial projective small deformation of any projective K3 surface, and when I with Shioda could complete explicit description of the Mordell-Weil lattices of rational elliptic surfaces. In the last two years, I was particularly interested in hyperkaehler manifolds (the most faithful generalization of K3 surfaces among Calabi-Yau manifolds in wider sense). If a hyperkaehler manifold admits a fibration over a normal projective variety, then it is necessarily Lagrangian (Matsushita). The general fibers are abelian varieties and if in addition it admits a bimeromorphic section, then the hyperkaehler manifold is also projective. It is conjectured that the base space is always a projective space. I classified general singular fibers (in the sense of complex geometry) with Hwang in a more general context, and I determined the Picard number of the generic fiber (in the sense of scheme) and derived the rank formula of the Mordell-Weil group, when it admits at least one holomorphic section over the projective space. Now, among other things, I am also interested in applications of these basic tools.
Ken'ichi OHSHIKA

**Topology**

My speciality is studying 3-manifolds, hyperbolic geometry and discrete groups. The study of 3-manifolds is originated from the work of Poincare, and has been pursued without interruption, being harnessed by a novel desire to understand the world in which we are living. At the turn of the present century, there was a big event, which was an affirmative resolution of the Thurston's geometrisation conjecture by Perelman. Still there remains a long way to go until we can say that we have completely understood 3-manifolds, and many topologists are actively studying them.

With the geometrisation conjecture being solved, the most important class of 3-manifolds is that of hyperbolic manifolds. Study of hyperbolic 3-manifolds is closely related to that of Kleinian groups, which has been done within the framework of complex analysis for a long time. I have been working in this field for a few decades.

The techniques and tools which were obtained in the field of hyperbolic geometry are now combined with geometric group theory, which was initiated by Gromov, and flourishing enormously. I am also interested in this field, and my target at the moment is to cultivate a new perspective combining my viewpoint of Kleinian group theory with geometric group theory.
I am mainly working on algebraic geometry, a subject in which various aspects of algebraic varieties are studied.

In the early days I was working on topics related to Geometric Invariant Theory (GIT) and birational geometry. GIT is a method of constructing quotients of algebraic varieties by algebraic group actions, and birational geometry, very roughly, deals with operations on algebraic varieties which only changes small parts of them. Constructions of GIT quotients require a choice of extra data, so called stability conditions. Different choices of stability conditions yield different quotients, and in good situations they are birationally equivalent. There are special cases in which the birational geometry of the quotients can be completely described in terms of GIT, and as a class of such quotient varieties the notion of Mori dream spaces was defined. I proved that images of morphisms from Mori dream spaces are again Mori dream spaces, and that the positivity of the canonical line bundle of a Mori dream space is closely related to the singularity of the variety on which the group acts.

These days I am working more on categorical view points on algebraic varieties. For example, I am studying so called semiorthogonal decompositions of the derived category of coherent sheaves on smooth projective minimal varieties. This is motivated by birational geometry, and its techniques are used in practice. Also I am working on moduli spaces of noncommutative algebraic varieties. This subject involves several interesting topics, such as derived category, GIT, elliptic curves, and birational transformations of noncommutative algebraic varieties.
In mathematics, we have several fields in which we try to describe how the things vary as time goes by, for example, differential equations, discrete dynamical systems (systems constructed by iteration of a map in a space), stochastic processes, etc. Related to those theories, there are various mathematical models in biology, economics, etc. For example, regarding the discrete dynamical systems theory, for a kind of insect, we have a model stating that the number of the insect in the \((n+1)\)-th year is the image of the number in the \(n\)-th year under a polynomial map \(f\) of degree two, where the map \(f\) does not depend on \(n\). In this model, to get a wider field of view, we sometimes consider initial values of complex number. Then, we have a discrete dynamical system in the complex plane, using the polynomial map \(f\). As the above, a system constructed by iteration of a polynomial (or holomorphic) map in the complex plane (or in a complex manifold) is called a “complex dynamical system”. To analyse such systems mathematically, we use complex analysis deeply. Regarding the iteration of a polynomial map \(f\) of degree two or greater in the complex plane, the set of initial values in the complex plane which has initial sensitivity (or chaotic behavior), which is called the “Julia set” for \(f\), is always not empty, and it has “self-similarity”. That is, if we magnify the detail of the Julia set, then it is similar to the whole Julia set. Recall that, we are very familiar with various sets having self-similarity, for example, clouds, trees, leaves, surfaces of mountains, etc. Those kinds of complicated (but beautiful) sets are called “fractal sets”, of which research was initiated by Mandelbrot in 1975. So, the Julia set of a polynomial map is one of typical examples of fractal sets. Regarding the complex dynamical systems in the complex plane, if we want to go further, we can use the theory of deformation of surfaces. By using it, we can know some information of the orbit of an initial value that does not have initial sensitivity. Going along this direction, you would notice that the theory of complex dynamical systems (of one variable) is very similar to that of discrete groups of linear fractional functions in the field of 2- or 3-dimensional geometry, which gives us much interest. I am a researcher of complex dynamical systems, and especially, I am one of pioneers of theory of dynamics of semigroups generated by polynomials (or rational functions). In this topic, we consider several polynomial maps \(f, g, h, \ldots\) and consider any images of initial values under the all maps \(f, g, h, \ldots\), and consider the images again, and we continue this procedure. In this setting, we can consider the following: (1) an estimate of “fractal dimension”, of which value may not be an integer, of the Julia set for a system, (2) random complex dynamical systems, in this direction we can get functions in the complex plane which are similar to the “devil’s staircase” (3) the investigation on how small copies of the whole Julia sets inside the Julia sets overlap; we use a kind of “cohomology theory” (a kind of algebra). Studying the complex dynamical systems, we can deal with both the real world in the nature and one of the deepest theory of mathematics, which is the fascination of this field.

My research field is Partial Differential Equations of hyperbolic and dispersive type. They arise in mathematical physics as equations describing wave propagation, so there are a wealth of applications and plenty of problems to be studied. Of my special interest is the nonlinear interactions of hyperbolic waves. Since the analysis of nonlinear PDE is still a developing subject, there are few general conclusions about that. To put it another way, it means that there are possibilities for coming across wonderful phenomena which no one has ever seen before.
My current interests are mathematical aspects of the superstring theory, in particular, algebraic geometry related to the mirror symmetry.

More precisely, I am studying homological algebras and moduli problems for categories of "D-branes" that extend derived categories of coherent sheaves on algebraic varieties.

Indeed, I am trying to construct Kyoji Saito's primitive forms and their associated Frobenius structures from triangulated categories defined via matrix factorizations attached to weighted homogeneous polynomials.

For example, I proved that the triangulated category for a polynomial of type ADE is equivalent to the derived category of finitely generated modules over the path algebra of the Dynkin quiver of the same type.

Now, I extend this result to the case when the polynomial corresponds to one of Arnold's 14 exceptional singularities and then showed the "mirror symmetry" between weighted homogeneous singularities and finite dimensional algebras, where a natural interpretation of the "Arnold's strange duality" is given.
Naohito TOMITA

Real Analysis

My research field is Fourier analysis, and I am particularly interested in the theory of function spaces. Fourier series were introduced by J. Fourier (1768-1830) for the purpose of solving the heat equation. Fourier considered as follows: "Trigonometric series can represent arbitrary periodic functions". However, in general, this is not true. Then, we have the following problem: "When can we write a periodic function as an infinite (or finite) sum of sine and cosine functions?". Lebesgue space which is one of function spaces plays an important role in this classical problem. Here Lebesgue space consists of functions whose p-th powers are integrable. In this way, function spaces are useful for various mathematical problems. As another example, modulation spaces were recently applied to pseudodifferential operators which are important tool for partial differential equations, and my purpose is to clarify their relation.

\[ \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \]

Motoo UCHIDA

Algebraic Analysis

My research field is algebraic analysis and microlocal analysis of partial differential equations. The viewpoint of microlocal analysis (with cohomology) is a new important point of view in analysis introduced by Mikio Sato in the early 1970s. Thinking from a microlocal point of view helps us to well understand a number of mathematical phenomena (at least for PDE) and to find a simple hidden principle behind them. Even for some classical facts (scattered as well-known results), we can sometimes find a new unified way of understanding from an algebro-analytic viewpoint.
Takao WATANABE

Number Theory

My current interest is the Geometry of Numbers. The Geometry of Numbers was founded by Hermann Minkowski in the beginning of the 20th century. Minkowski proved a famous theorem known as "Minkowski's convex body theorem", which asserts that "there exists a non-zero integer point in V if V is an o-symmetric convex body in the n-dimensional Euclidean space whose volume is greater than $2^n$". When V is an ellipsoid, this theorem is refined as follows. Let A be a non-singular 3 by 3 real matrix and K(c) the ellipsoid consisting of points x such that the inner product (Ax, Ax) is less than or equal to c > 0. For i = 1, 2, 3, we define the constant $c_i$ as the minimum of c > 0 such that K(c) contains i linearly independent integer points. Then $c_1$, $c_2$, $c_3$ satisfies the inequality $c_1c_2c_3 <= 2\text{det } A^n/2$. This is called "Minkowski's second theorem". A similar inequality holds for any n-dimensional ellipsoid. Namely, if A is a non-singular n by n real matrix and K(c) is the n-dimensional ellipsoid defined by (Ax, Ax) <= c, we can define $c_i$ for i = 1, 2, ..., n as the minimum of c > 0 such that K(c) contains i linearly independent integer points. Then the inequality $c_1c_2...c_n <= h(n)\text{det } A^n/2$ holds for any A. The optimal upper bound h(n) does not depend on A, and is called Hermite's constant. We know h(2) = 4/3, h(3) = 2, h(4) = 4, ..., h(8) = 256, but h(n) for a general n is not known. A recent major topic of this research area is the determination of h(24). In 2003, Henry Cohn and Abhinav Kumar proved that h(24) = 4/24. (Incidentally, h(3) was essentially determined by Gauss in 1831, and h(8) was determined by Blaschke in 1953. If you would determine h(9), then your name would be recorded in treatises on the Geometry of Numbers.) Now I study (an analogue of) the Geometry of Numbers on algebraic homogeneous spaces. One of my results is a generalization of Minkowski's second theorem to a Severi-Beraner variety. In addition, I am interested in the reduction theory of arithmetic subgroups, automorphic forms, the algebraic theory of quadratic forms and Diophantine approximation.

Katsutoshi YAMANOI

Complex Geometry

My research interest is Complex geometry and Complex analysis, both from the viewpoint of Nevanlinna theory. In the geometric side, I am interested in the conjectural second main theorem in the higher dimensional Nevanlinna theory for entire holomorphic curves into projective manifolds. Also I am interested in the behavior of Kobayashi pseudo-distance of projective manifolds of general type. These problems are related to an algebraic geometric problem of bounding the canonical degree of algebraic curves in projective manifolds of general type by the geometric genus of the curves. In the analytic side, I am interested in classical problems of value distribution theory for meromorphic functions in the complex plane.
Seidai YASUDA

I am interested in the special values of L-functions, especially Hasse-Weil L functions. Hasse-Weil L-functions are defined for arithmetic schemes or motives by using geometric cohomology. I also study multiple zeta values.

Systems of polynomials with integral coefficients are studied in number theory. It is often very difficult to find the integral solutions of such a system. Instead, we simultaneously deal with the solutions in various commutative rings. The solutions in various rings forms a scheme, which provides geometric methods for studying the system of polynomials.

Special values of Hasse-Weil L functions are believed to be related to motivic cohomologies, which are defined by using algebraic cycles or algebraic K-theory and are usually hard to know explicitly. It is a very deep prediction which relates abstract objects with the concrete objects, L-functions. Multiple zeta values are related not only to the fundamental groups of moduli spaces of genus zero curves, mixed Tate motives, but also KZ equation, and Drinfeld associators.

It is expected that motives are related to automorphic representations. The expectation is important since we have various methods for studying automorphic L-functions. Some relations between motives and automorphic representations are realized by using Shimura varieties. In a joint work with Satoshi Kondo, I have proved a equality relating motivic cohomologies and special values of Hasse-Weil L-functions for some function field analogues of Shimura varieties.

Hasse-Weil L-functions are defined via some Galois representations. We need to study such Galois representations. For some technical reasons it is important to study Galois representations of p-adic fields with p-adic coefficients, and p-adic Hodge theory provides some tools for studying such representations. For recent years there have been much development in p-adic Hodge theory, and a lot of beautiful theories have been constructed. However the theory is not fully established and many aspects of the theory remains mysterious. I am now trying to make the integral p-adic Hodge theory more convenient for explicit computation.

Takehiko YASUDA

My main research object is singularities of algebraic varieties. An algebraic variety is a "figure" formed by solutions of algebraic equations. Such a figure often has points where the figure is sharp-pointed or intersects itself. Singularities make the study of an algebraic variety difficult. However since they often appear under various constructions, it is important to study them. Also singularities are interesting research object themselves.

More specifically, I am interested in resolution of singularities, the birational-geometric aspect of singularities, the McKay correspondence. Although these are classical research areas, changing a viewpoint or the setting of a problem, one can sometimes find a new phenomenon. Such a discovery is the greatest pleasure in my mathematical research.

To pursue research, I use various tools like motivic integration, Frobenius maps, moduli-theoretic blowups, non-commutative rings, and sometimes make ones by myself.

Recently I am fascinated by mysterious behaviors of singularities in positive characteristic (a world where summing up several 1's gives 0.)
In our research group of topology, we hold four kinds of specialised seminars regularly: the low-dimensional topology seminar focusing on the knot theory, three-manifolds, and hyperbolic geometry (Ken’ichi Ohshika, Hideki Miyachi); the differential topology seminar focused on contact structures, symplectic structures, foliations, and dynamical systems (Kenji Yamato); the seminar on transformation groups (Yasuhiro Hara); and the seminar on four-manifolds focused on 4-manifolds and complex surfaces (Kazunori Kikuchi). We also hold a monthly topology seminar encompassing all fields of topology, where all of us meet together.
Seminar of Differential Equations

Our seminar is held every Friday from 15:30 to 17:00. One of the features of the seminar is to cover a wide variety of topics on Qualitative Analysis of Differential Equations. In fact, we are interested in ordinary differential equations, partial differential equations, linear differential equations, nonlinear differential equations and so on. Lecturers are invited from not only domestic universities, but also foreign countries and present us their original results or survey of recent development of their fields. Furthermore, this seminar provides opportunities to give a talk for our colleagues and Ph.D. students majoring in differential equations. Moreover, we should mention that we are pleased to have participants from other universities located close to ours. In this way we communicate with each other and try to contribute to the progress of the theory of differential equations.

Seminar on Probability

"Seminar on Probability" is held Tuesday evenings by probabilists belonging to the graduate school of science and the graduate school of engineering science. The range of the seminar topics are the following:

(1) **Probability theory**;

Stochastic analysis and infinite dimensional analysis (problems arising from other areas of mathematics such as real analysis, differential equations and differential geometry);

(2) **Problems related to the probability theory**

arising from statistical physics, stochastic control theory and mathematical finance.

We enjoy visits and talks by many researchers from other universities, domestic and abroad, and wish this seminar to continue stimulating research interaction.

Dynamics and Fractals Seminar

Researchers and students working on various fields related to dynamical systems and fractals attend this seminar. We meet once a month for approximately 90 minutes. Each talk on his/her research is followed by discussions among all participants.