

Department of Mathematics

Summary

Mathematics, originally centered in the concepts of number, magnitude, and form, has long been growing since ancient Egyptian times to the 21st century. Through the use of abstraction and logical reasoning, it became an indispensable tool not only in natural sciences, but also in engineering and social sciences. Recently, the remarkable development of computers is now making an epoch in the history of mathematics.

Department of Mathematics is one of the six departments of Graduate School of Science, Osaka University. It consists of 6 research groups, all of which are actively involved in the latest developments of mathematics. Our mathematics department has ranked among the top seven in the country.

The department offers a program with 32 new students enrolled annually leading to post-graduate degrees of Masters of Science. The department also offers a Ph.D. program with possibly 16 new students enrolled annually.

Graduate courses are prepared so as to meet various demands of students. Besides introductory courses for first year students, a number of topics courses are given for advanced students. Students learn more specialized topics from seminars under the guidance of thesis advisors.

Our department has our own library equipped with about 500 academic journals and 50,000 books in mathematics, both of which graduate students can use freely. Also by an online system, students as well as faculty members can look up references through Internet.

Research Groups

Algebra, Geometry, Analysis, Global Geometry & Analysis, Experimental Mathematics, Mathematical Science.

Areas of Research

Number Theory, Ring Theory, Algebraic Geometry, Algebraic Analysis, Partial Differential Equations, Real Analysis, Differential Geometry, Complex Differential Geometry, Topology, Knot Theory, Discrete Subgroups, Transformation Groups, Complex Analysis, Complex Functions of Several Variables, Complex Manifolds, Discrete Mathematics, Probability

Theory, Dynamical Systems, Fractals, Mathematical Engineering, Information Geometry, Mathematical Physics.

Faculty Members

Professors (16)

Shin-ichi DOI, Osamu FUJINO, Akio FUJIWARA, Ryushi GOTO, Masashi ISHIDA, Seiichi KAMADA, Soichiro KATAYAMA, Takehiko MORITA, Hiroaki NAKAMURA, Shin-ichi OHTA, Hiroshi SUGITA, Atsushi TAKAHASHI, Naohito, TOMITA, Takao WATANABE, Katsutoshi YAMANOI, Takehiko YASUDA.

Associate Professors (13)

Shinpei BABA, Ichiro ENOKI, Kento FUJITA, Takahisa INUI, Hisashi KASUYA, Eiko KIN, Haruya MIZUTANI, Tomonori MORIYAMA, Tadashi OCHIAI, Mamoru OKAMOTO, Shinnosuke OKAWA, Yuichi SHIOZAWA, Motoo UCHIDA, .

Lecturer (1)

Kazunori KIKUCHI.

Assistant Professors (8)

Yasuhiro HARA, Takao IOHARA, Erika KUNO, Tatsuki KUWAGAKI, Yoshihiko MATSUMOTO, Hiroyuki OGAWA, Koji OHNO, Kenkichi TSUNODA.

Cooperative Members in Osaka University

Professors (7)

Susumu ARIKI, Daisuke FURIHATA, Takayuki HIBI, Katsuhisa MIMACHI, Yoshie SUGIYAMA, Katsuhiko UNO, Masaaki WADA.

Associate Professors (6)

Tsuyoshi CHAWANYA, Akihiro HIGASHITANI, Yuto MIYATAKE, Norio NAWATA, Yoshiki OSHIMA, Kouichi YASUI.

Home Page

<http://www.math.sci.osaka-u.ac.jp/eng/>

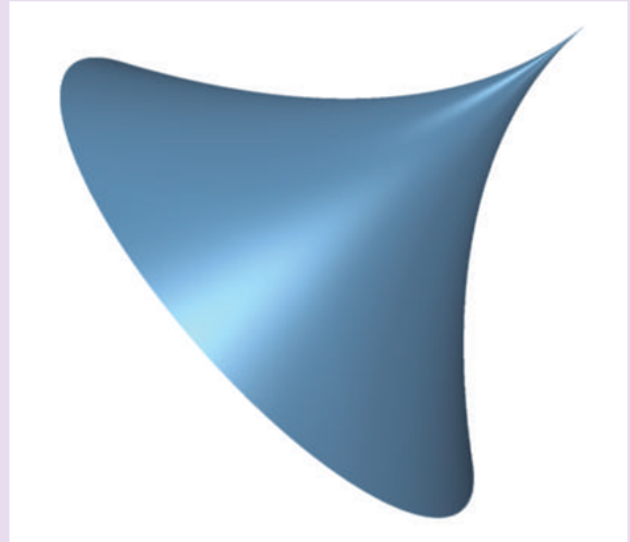
Shinpei BABA

Low-dimensional Geometry, Topology

My research is centered on surfaces (i.e. 2-dimensional manifolds), which are fundamental objects in geometry and topology.

In particular, I am interested in the relations between geometric structures (locally homogeneous structures) on surfaces, and representations of the fundamental groups of surfaces S (surface groups) into Lie groups G .

In the case that the Lie groups $G = \mathrm{PSL}(2, \mathbb{R})$ or $\mathrm{PSL}(2, \mathbb{C})$ and that representations have discrete images, beautiful theories have been developed extensively, in particular, in relation to the classification and the deformation theorem of 2- and 3-dimensional manifolds.



Shin-ichi DOI

Partial Differential Equations

Partial differential equations have their origins in various fields such as mathematical physics, differential geometry, and technology. Among them I am particularly interested in the partial differential equations that describe wave propagation phenomena: hyperbolic equations and dispersive equations. A typical example of the former is the wave equation, and that of the latter is the Schrödinger equation. For many years I have studied basic problems for these equations: existence and uniqueness of solutions, structure of singularities of solutions, asymptotic behavior of solutions, and spectral properties. Recently I make efforts to understand how the singularities of solutions for Schrödinger equations or, more generally, dispersive

equations propagate. The center of this problem is to determine when and how the singularities of solutions for the dispersive equations can be described by the asymptotic behavior of solutions for the associated canonical equations.

Ichiro ENOKI

Complex Differential Geometry

A complex manifold is, locally, the world build out of open subsets of complex Euclid spaces and holomorphic functions on them. If two holomorphic functions are defined on a connected set and coincide on an open subset, then they coincide on the whole. Complex manifolds inherit this kind of property from holomorphic functions. That is, they are stiff and hard in a sense. It seems to me that complex manifolds are not metallicly hard but have common warm feeling with wood or bamboo, which have grain and gnarl. Analytic continuations, as you learned in the course on the function theory of complex variables, is analogous to the process of growth of plants. Instead of considering whole holomorphic functions, a class of complex manifold can be build out of polynomials. This is the world of complex algebraic manifolds, the most fertile area in the world of complex manifolds. To complex algebraic manifolds, since they are algebraically defined, algebraic methods are of course useful to study them. In certain cases, however, transcendental methods (the word "transcendental"

means only "not algebraic") are powerful. For example, one of the simplest proof for the fundamental theorem of algebra is given by the function theory of one complex variable. These two methods have been competing with each other since the very beginning of the history of the study of complex manifolds. This competing seems to me the prime mover of the development of the theory of complex manifolds. Comparing the world of complex manifold to the earth, the world of complex algebraic manifold is to compare to continents, and the boundary to continental shells. The reason I wanted to begin to study complex manifolds was I heard the Kodaira embedding theorem, which characterizes complex algebraic manifolds in the whole complex manifolds. The place I begin to study is, however, something like the North Pole or the Mariana Trench. Now the center of my interest is in the study of complex algebraic manifolds by transcendental methods. (Thus I have reached land but I found this was a jungle.)

Osamu FUJINO

Algebraic Geometry

I am mainly interested in algebraic geometry. More precisely, I am working on the birational geometry of higher-dimensional algebraic varieties. In the early 1980s, Shigefumi Mori initiated a new approach for higher-dimensional birational geometry, which is now usually called the Minimal Model Program or Mori theory. Unfortunately, this beautiful approach has not been completed yet. One of my dreams is to complete the Minimal Model Program in full generality. I am also interested in toric geometry, Hodge theory, complex geometry, and so on.



Department
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Kento FUJITA

Algebraic Geometry

My research interest is algebraic geometry. Especially, I am interested in birational behavior of Fano varieties, a special class of algebraic varieties. I have researched the Mukai conjecture, the existence of certain good models over some reducible varieties, and an algorithm to classify log del Pezzo surfaces (joint work with Kazunori Yasutake), etc. Recently, I am interested in K-stability of Fano varieties; I (and independently Chi Li) gave a birational interpretation of K-stability of Fano varieties.

Department
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Mathematics

Akio FUJIWARA

Mathematical Engineering

"What is information?" Having this naive yet profound question in mind, I have been working mainly on noncommutative statistics, information geometry, quantum information theory, and algorithmic randomness theory.

One can regard quantum theory as a noncommutative extension of the classical probability theory. Likewise, quantum statistics is a noncommutative extension of the classical statistics. It aims at finding the best strategy for identifying an unknown quantum object from a statistical point of view, and is one of the most exciting research field in quantum information science.

Probability theory is usually regarded as a branch of analysis. Yet it is also possible to investigate the space of probability measures

from a differential geometrical point of view. Information geometry deals with a pair of affine connections that are mutually dual (conjugate) with respect to a Riemannian metric on a statistical manifold. It is known that geometrical methods provide us with useful guiding principle as well as insightful intuition in classical statistics. I am interested in extending information geometrical structure to the quantum domain, admitting an operational interpretation.

I am also delving into algorithmic and game-theoretic randomness from an information geometrical point of view. Someday I wish to reformulate thermal/statistical physics in terms of algorithmic information theory.

Ryushi GOTO

Geometry

My research interest is mostly in complex and differential geometry, which are closely related with algebraic geometry and theoretical physics. My own research started with special geometric structures such as Calabi-Yau, hyperKähler, G_2 and $Spin(7)$ structures. These four structures exactly correspond to special holonomy groups which give rise to Ricci-flat Einstein metrics on manifolds. It is intriguing that these moduli spaces are smooth manifolds on which local Torelli type theorem holds. In order to understand these phenomenon, I introduce a notion of geometric structures defined by a system of closed differential forms and establish a criterion of unobstructed deformations of structures. When we apply

this approach to Calabi-Yau, hyperKähler, G_2 and $Spin(7)$ structures, we obtain a unified construction of these moduli spaces. At present I also explore other interesting geometric structures and their moduli spaces.

Yasuhiro HARA

Topology

The field of my study is topology and, especially, I study the theory of transformation groups. The Borsuk-Ulam theorem is one of famous theorems about transformation groups. This theorem is often taken up as an application in elementary lectures about the homology theory. The content of the theorem is as follows: for every continuous map from the n -dimensional sphere to the n -dimensional Euclidean space, there exists a point such that the map takes the same value at the point and at the antipodal point. A famous application of this theorem is the following. "On the earth, there is a point such that the temperature and humidity at the point are the same as those at the antipodal point." We consider a free

action of a group of order two on the n -dimensional sphere to prove the Borsuk-Ulam theorem. Then for any equivariant map (any continuous map which preserves the structure of the group action) from the sphere to itself, the degree of the map is odd. By using this fact, we obtain the Borsuk-Ulam theorem. In the case of the Borsuk-Ulam theorem, we consider spheres and free actions of a group of order two. Actually, when we consider other manifolds and actions of other groups, there are some restrictions of homotopy types of equivariant maps. I study such restrictions of homotopy types of equivariant maps by using the cohomology theory, and I study relationships between homotopy types of equivariant maps and topological invariants.

Takahisa INUI

Nonlinear Partial Differential Equations

My research subject is nonlinear partial differential equations. Especially, I am interested in the global behavior of the solutions to nonlinear dispersive equations or nonlinear wave equations.

Dispersive equations describe the dispersion phenomena of waves and are basic equations in quantum physics. For example, Schroedinger equation and Klein--Gordon equation are typical dispersive equations, which appear in quantum physics or relativistic quantum field theory. Wave equations express the properties of motion in waves. Considering nonlinear interactions between particles,

we can treat various physical phenomena, for example, optics, superconductivity, and

Bose-Einstein condensate, by the nonlinear dispersive or wave equations. Nonlinear dispersive or wave equations have two properties.

One is dispersion and the other is nonlinearity. They conflict each other. Thus, there are so many behaviors of the solutions. I study this subject to find all behavior.

Takao IOHARA

Nonlinear Partial Differential Equations

My research interest is concerned with nonlinear partial differential equations appearing in fluid mechanics. The current research topic is the equations of the motion of viscous incompressible fluid which has free moving surface. The motion of viscous incompressible fluid is governed by the Navier-Stokes equations, which are not easy to solve because of their nonlinearity. The free moving surface adds another nonlinearity to the problem and the study of it needs more elaborate technique than the problems on fixed domain.



Masashi ISHIDA

Differential Geometry

My research interest is in geometry, particularly, interaction between topology and differential geometry. For instance, I am studying the nonexistence problems of Einstein metrics and Ricci flow solutions on 4-manifolds by using the Seiberg-Witten equations. I am also interested in the geometry of the Yamabe invariant. The computation of the Yamabe invariant for a given manifold is a difficult problem in general. By using the Seiberg-Witten equations, I determined the exact value of the Yamabe invariant for a large class of 4-manifolds which includes complex surfaces as special cases. Furthermore, I am also interested in both the Ricci flow in higher dimension and some generalized versions of the Ricci flow like the

Ricci Yang-Mills flow. The Ricci flow was first introduced by R. Hamilton in 1981 and used as the main tool in G. Perelman's solution of the Poincare conjecture in 2002. Perelman introduced many new and remarkable ideas to prove the conjecture. The theory developed by Hamilton and Perelman is now called the Hamilton-Perelman theory. One of my recent interests is to investigate geometric analytical properties of the generalized versions of the Ricci flow from the Hamilton-Perelman theoretical point of view.

Seiichi KAMADA

Topology

My research area is topology (geometric topology-- knot theory, low dimensional topology). I have been working on surface-links in 4-space, including 2-knots and 2-links, 2-dimensional braids, braided surfaces, and 4-manifolds.

Surface-links are closed surfaces smoothly embedded in the 4-space, and they are considered equivalent if they are ambiently isotopic. It is a quite difficult problem to decide whether two given surface-links are equivalent or not. Invariants are often used to show that two surface-links are not equivalent. However, few invariants are known for surface-links. I am interested in studying surface-links and their invariants using "quandles" and graphics so-called

"charts".

A quandle is an algebra with a binary operation which satisfies three axioms corresponding to Reidemeister moves in knot theory. A homology theory of quandles was established and we can use it for construction of invariants for surface-links. 2-dimensional braids, or surface braids, are a generalization of classical braids. They are in one-to-one correspondence to oriented surface-links in 4-space modulo certain basics moves. As a method of describing 2-dimensional braids, a chart description was introduced. This method has been extended in a general theory of charts. For example, charts can be used for studies of Lefschets fibrations of 4-manifolds.

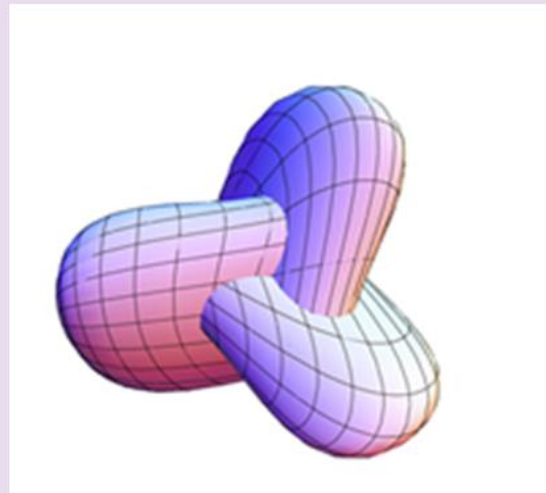
Hisashi KASUYA

Geometry

Until now, I have tried to extend the geometry of nilpotent groups to the geometry of solvable groups. More precisely, I have studied the cohomology theory of homogeneous spaces of solvable Lie groups and complex geometry of non-Kähler manifolds. It seems that the gap between nilpotent groups and solvable groups is small. But this gap contains a potential for geometry. By the growing out of left-invariance and non-triviality of local system cohomology, I succeeded in giving a great surprise.

Recently, I am interested in the geometry which relates to reductive or semi-simple groups in contrast to nilpotent or solvable groups. In particular, I study non-abelian Hodge theory, variations of hodge structures,

lattices in semi-simple Lie groups and locally homogeneous spaces.



Soichiro KATAYAMA

Nonlinear Partial Differential Equations

My research interest is in nonlinear partial differential equations. To be more specific, I am working on the initial value problem for nonlinear wave equations (in a narrow sense), and also for partial differential equations describing the nonlinear wave propagation in a wider sense, such as Klein-Gordon equations and Schroedinger equations.

The initial value problem is to find a solution to a given partial differential equation with a given state at the initial time (a given initial value). However, in general, it is almost impossible to give explicit expression of solutions to nonlinear equations. Therefore, in the mathematical theory, it is important to investigate the existence of solutions and also their behavior when they

exist.

If we consider the initial value problem for the equations mentioned above and if the initial value is sufficiently small, the existence of solutions up to arbitrary time (the existence of global solutions) is mainly determined by the power of the nonlinearity. Especially, when the nonlinearity has the critical power, the existence and non-existence of global solutions depend also on the detailed structure of the nonlinear terms. I am interested in this kind of critical case, and studying sufficient conditions for the existence of global solutions and their asymptotic behavior.

Kazunori KIKUCHI

Differential Topology

I have been studying topology of smooth four-dimensional manifolds, in particular interested in homology genera, representations of diffeomorphism groups to intersection forms, and branched coverings. Let me give a simple explanation of what interests me the most, or homology genera. The homology genus of a smooth four-dimensional manifold M is a map associating to each two-dimensional integral homology class $[x]$ of M the minimal genus g of smooth surfaces in M that represent $[x]$. For simplicity, reducing the dimensions of M and $[x]$ to the halves of them respectively, consider as a two-dimensional manifold the surface of a doughnut, or torus T , and a one-dimensional integral homology class $[y]$ of T . Draw a meridian and a longitude on T as on the terrestrial sphere, and let $[m]$ and $[l]$ denote the homology classes of T represented by the meridian and the longitude respectively. It turns out that $[y] = a[m] + b[l]$ for some integers a and b , and that $[y]$ is

represented by a circle immersed on T with only double points. Naturally interesting then is the following question: what is the minimal number n of the double points of such immersions representing $[y]$? Easy experiments would tell you that, for example, $n = 0$ when $(a,b) = (1,0)$ or $(0,1)$ and $n = 1$ when $(a,b) = (2,0)$ or $(0,2)$. In fact, it is proved with topological methods that $n = d-1$, where d is the greatest common divisor of a and b . It is the minimal number n for T and $[y]$ that corresponds to the minimal genus g for M and $[x]$. The study on the minimal genus g does not seem to proceed with only topological methods; it sometimes requires methods from differential geometry, in particular methods with gauge theory from physics; though more difficult, it is more interesting to me. I have been tackling the problem on the minimal genus g with such a topological way of thinking as to see things as if they were visible even though invisible.

Eiko KIN

Topology, Dynamical Systems

I am interested in the mapping class groups on surfaces. The most common elements in the mapping class groups are so called pseudo-Anosovs. I try to understand which pseudo-Anosovs are the most simple in the mapping class groups. I describe my goal more clearly. Pseudo-Anosovs possess many complicated (and beautiful) properties from the view points of the dynamical systems and the hyperbolic geometry. There are some quantities which reflect those complexities of pseudo-Anosovs. Entropies and volumes (i.e., volumes of mapping tori) are examples. We fix the topological type of the surface and we consider the set of entropies (the set of volumes) coming from the pseudo-Anosov elements on the surface. Then one can see that there exists a minimum of the set. That is, we can talk about the pseudo-Anosovs with the minimal entropies (pseudo-Anosovs with the minimal volumes). I would like to know which pseudo-Anosov achieves the minimal entropy with the minimal volume. Recently, Gabai, Meyerhoff and Milley determined hyperbolic

closed 3-manifolds and hyperbolic 3-manifolds with one cusp with very small volume. Intriguingly, the result implies that those hyperbolic 3-manifolds are obtained from the single hyperbolic 3-manifold by Dehn filling. Some experts call the single 3-manifold the "magic manifold". Said differently, the magic manifold is a parent manifold of the hyperbolic manifolds with very small volume. It seems likely that we have the same story in the world of pseudo-Anosovs with the very small entropies. This conjecture is based on the recent works of myself and other specialists. We note that there are infinitely many topological types of surfaces (for example, the family of closed orientable surface with genus g). For the mapping class group of each surface we know that there exists a pseudo-Anosov element with the minimal entropy. Thus, of course, there exist infinitely many pseudo-Anosov elements with the minimal entropies. It might be true that all minimizers are obtained from the magic 3-manifold. When I work on this project, I sometimes think of our universe.

Erika KUNO

Topology

My research theme is mainly mapping class groups of surfaces (including non-orientable surfaces) and 3-dimensional handlebodies. In particular, I am interested in exploring them from the viewpoints of geometric group theory. Geometric group theory is a new field among a lot of areas of mathematics and it is progressing significantly. One of the most important problems in geometric group theory is classifying finitely generated groups by “quasi-isometries”. Two finitely generated groups are quasi-isometric if roughly speaking, their word metrics are the same up to linear functions. An interesting part of the geometric group theory is that the properties of the infinite groups are revealed one by one by measuring with a coarse scale of quasi-

isometries, but not isometries. Currently, groups which are quasi-isometric to mapping class groups have hardly been found. Then what I am wondering is the question “Which groups are quasi-isometric to the mapping class group?”. Based on this big theme, I would like to elucidate properties of mapping class groups, and deepen their understanding.

Tatsuki KUWAGAKI

Mathematical Physics, Geometry, Algebraic Analysis

Algebraic analysis is an algebraic/geometric study of analysis. On the other hand, symplectic geometry originated to treat classical mechanics by using geometry. These two mathematics are related to each other, and now it is popular to study symplectic geometry in terms of algebraic analysis. I am particularly interested to solve symplectic geometric problems arising from mathematical physics by using algebraic analysis. For example, mirror symmetry phenomena (which is a conjecture of string theory) predicts an equivalence between symplectic geometric categories and algebraic geometric categories. The symplectic categories resemble to categories formed by differential equations, and one can study algebraic analytically.

I am using this idea to study mirror symmetry, or conversely, study algebraic analytical problems (Riemann-Hilbert correspondence and WKB analysis) in terms of symplectic geometry.

Yoshihiko MATSUMOTO

Differential Geometry, Several Complex Variables

Working on differential geometry, partly with some flavor of function theory of several complex variables. I've been mainly studying geometry of "asymptotically complex hyperbolic spaces," with emphasis on a partial differential equation called Einstein's equation on them. While being a generalization of geometry of bounded strictly pseudoconvex domains in function theory of several complex variables, it's beyond the scope of the field of complex geometry. Based on this experience, I'm now aiming toward some more general theory that applies to other "spaces that converge to ones with much symmetry," which are called "asymptotically symmetric spaces."

Asymptotically hyperbolic spaces, which are the most basic examples of asymptotically symmetric spaces, are lacking symmetries such as the homogeneity or the isotropicity of the genuine hyperbolic spaces in the strict sense. However, as a point in the space moves more away from a fixed one, its neighborhood looks more like an open set of the hyperbolic space. Recall the "natural" conformal structure on the sphere at infinity of the hyperbolic space—the boundary at infinity of an asymptotically hyperbolic space is equipped with a conformal structure in the same way. In the case of asymptotically complex hyperbolic spaces, whose model has

little less isotropicity than that of the hyperbolic space, the associated geometric structure on the boundary at infinity is the CR (Cauchy–Riemann) structure.

The fundamental question of geometry of asymptotically symmetric spaces is the following: what property of the space reflects the geometric structure at infinity and the topology of the space, and in what way? This includes the question whether or not there is a space with some certain property under a given condition on the structure at infinity and the topology. The existence problem of Einstein metrics is a typical example.

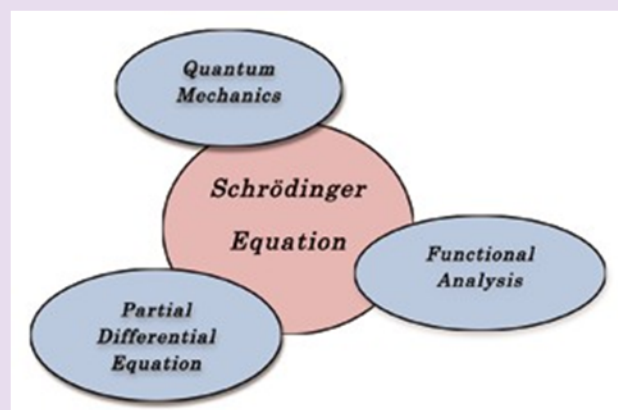
Although asymptotically hyperbolic spaces have been studied for several decades, many fundamental questions remain unsolved. And, if we think of understandings from the general viewpoint of geometry of asymptotically symmetric spaces, our field still seems to be kind of a wilderness. I'd cultivate it by going back and forth between analyses on special cases and abstract considerations.

Haruya MIZUTANI

Partial Differential Equations

The Schrödinger equation is the fundamental equation of physics for describing quantum mechanical behavior. I am working on the mathematical theory of the Schrödinger equation and my research interest includes scattering theory, semiclassical analysis, spectral theory, geometric microlocal analysis and so on. My current research has focused on various estimates such as decay or Strichartz inequalities, which describe dispersive or smoothing properties of solutions and are fundamental for studying linear and nonlinear dispersive equations. In particular, I am interested in understanding quantitatively the influence of the geometry of associated classical mechanics on the

behavior of quantum mechanics, via such inequalities.



Takehiko MORITA

Ergodic Theory, Probability Theory, Dynamical System

I specialize in ergodic theory. To be more precise, I am studying statistical behavior of dynamical systems via thermodynamic formalism and its applications.

Ergodic theory is a branch of mathematics that studies dynamical systems with measurable structure and related problems. Its origins can be found in the work of Boltzmann in the 1880s which is concerned with the so called Ergodic Hypothesis. Roughly speaking the hypothesis was introduced in order to guarantee that the system considered is ergodic i.e. the space averages and the long time averages of the physical observables coincide. Unfortunately, it turns out that dynamical systems are not always ergodic in general. Because of such a

background, the ergodic problem (= the problem to determine a given dynamical system is ergodic or not) has been one of the important subjects since the theory came into existence. In nowadays ergodic theory has grown to be a huge branch and has applications not only to statistical mechanics, probability, and dynamical systems but also to number theory, differential geometry, functional analysis, and so on.

Tomonori MORIYAMA

Number Theory

I am interested in automorphic forms of several variables. A classical automorphic (modular) form of one variable is a holomorphic function on the upper half plane having certain symmetry. Such functions appear in various branches of mathematics, say notably number theory, and have been investigated by many mathematicians.

There is a family of manifolds called Riemannian symmetric spaces, which is a higher-dimensional generalization of the upper half plane. The set of isometries of a Riemannian symmetric space forms a Lie group G . Roughly speaking, an automorphic form of several variables is a function on a Riemannian symmetric space satisfying the relative invariance under an "arithmetic"

subgroup of G and certain differential equations arising from the Lie group G . Studies on automorphic forms of several variables started from C. L. Siegel's works in 1930s and have been developed through interaction with mathematics of the day.

Currently I am working on two themes: (i) the zeta functions attached to automorphic forms and (ii) explicit constructions of automorphic forms, by employing representation theory of reductive groups over local fields. One of the joy in studying this area is to discover a surprisingly simple structure among seemingly complicated objects.

Hiroaki NAKAMURA

Number Theory

Theory of equations has a long history of thousands of years in mathematics, and, passing publication of the famous Cardano-Ferrari formulas in Italian Renaissance, Galois theory in the 19th century established a necessary and sufficient condition for an algebraic equation to have a root solution in terms of its Galois group. My research interest is a modern version of Galois theory, especially its arithmetic aspects. In the last century, the notion of Galois group was generalized to "arithmetic fundamental group" by Grothendieck, and Belyi's discovery (of an intimate relationship between Galois groups of algebraic numbers and fundamental groups of topological loops on hyperbolic curves) undertook a new area

of "anabelian geometry". Here are important problems of controlling a series of covers of algebraic curves and their moduli spaces, and Ihara's theory found deep arithmetic phenomena therein. Related also to Diophantus questions on rational points, fields of definitions and the inverse Galois problem, nowadays, there frequently occur important developments as well as new unsolved problems. I investigate these topics, and hope to find new perspectives for deeper understanding of the circle of ideas.

Tadashi OCHIAI

Arithmetic Geometry

I study Number theory and Arithmetic Geometry. In the research of Number theory, we study not only properties of integers and rational numbers, but every kinds of problems related to integers. For example, we are very much interested in rational points of algebraic varieties defined over the field of rational numbers. Number theory has a long history and we had a great progress, which is typical in the proof of Fermat's conjecture by Wiles and the proof of Mordell's conjecture by Faltings in 20th century. At the beginning of my career, my subject of research was the study of the l -adic étale cohomology of varieties over local fields. More recently, I am interested in the study of special values of zeta functions via the philosophy of Iwasawa

theory. Precisely speaking, my project is to study Iwasawa theory from the view point of Galois deformations. I think that Number theory is full of surprise as we see a lot of unexpected relations between different kinds of objects.

Hiroyuki OGAWA

Algebraic Number Theory

I have an interest in periodic objects. Expanding rational numbers into decimal numbers is delightful. The decimal number expansion becomes the repeat of a sequence of some integers. I have an appetite for continued fraction expansions, never get tired to calculate it, and want to find continued fractions with sufficiently long period. It is on the way to Gauss' class number one conjecture. Recently, I am studying iteration of rational functions. For a rational function $g(x)$ with rational coefficients, a complex number z with $g(g(\dots g(z)\dots))=z$ is called a periodic point on $g(x)$ and is an algebraic number. I expect that number theoretical properties which such an algebraic number z has is described by the rational function $g(x)$.

This does not seem to work out anytime, but one can find many rational functions $g(x)$ that describe the Galois group, the class number, the class group, and so on of a periodic point of $g(x)$. I think that this should be surely useful, and calculate like these every day.

Koji OHNO

Algebraic Geometry

When I was a student, I thought I knew number theory, geometry, but algebraic geometry was unfamiliar for me. One may say algebra and geometry are different fields, but you know the theory of quadratic curves and are aware of efficiency of algebraic methods for solving geometric problems. The field called algebraic geometry lies on such a line. When I was studying the theory of quadratic curves, I wondered, "Why do they only treat special equations like quadratics? There are many other equations. But how can they be treated?". When I discovered the answer might lie on this field, I decided to enter this field. The easiest non-trivial equation has the form such as "the second power of $y =$ an equation of x of degree three", which defines the so called "an elliptic curve". The theory of elliptic curve was one of the greatest achievements of nineteenth century and keeps developing today. Recently, the famous Fermat's

conjecture has been solved using this theory. The theory of quadratic and elliptic curves involve only two variables x, y . It is natural to think of the equations with many variables. In fact the algebraic geometers are expanding the theory, curves to surfaces and higher dimensional cases these days. Two dimensional version of elliptic curves are called K3 surfaces, which can be treated only using the theory of linear algebra(!) thanks to the Torelli's theorem. These days, the 3-dimensional versions, which is called Calabi-Yau threefolds are fascinating for algebraic geometers like me. Somehow theoretical physicists are also interested in this field. To study Calabi-Yaus by specializing these to ones with a fiber structure (on which field, I'm now working) might be one method, but I have been thinking that a new theory is needed. These days, many intriguing new theories have appeared and one may find more!

Shin-ichi OHTA

Differential Geometry, Geometric Analysis

My research subject is geometry, especially differential geometry and geometric analysis related to analysis and probability theory. A keyword of my research is “curvature” which represents how the space is curved. As seen in the difference between the sums of interior angles of triangles in a plane and a sphere, the behavior of the curvature influences various properties of the space (the shapes of triangles, the volume growth of concentric balls, how heat propagates, the behavior of entropy, etc.). This powerful and versatile conception has been applied to Riemannian manifolds, metric spaces, Finsler manifolds, Banach spaces, as well as discrete objects such as graphs.



Mamoru OKAMOTO

Nonlinear partial differential Equations

My research interest is in nonlinear partial differential equations. Especially, I am working on the Cauchy problem for nonlinear dispersive equations, such as nonlinear Schrödinger and nonlinear wave equations. The Cauchy problem is a problem to find a solution to a differential equation satisfying what are known as initial conditions (initial data) at a certain (initial) time.

For nonlinear problems, it is usually not possible to obtain explicit solutions, so we need to show an existence result of a solution. It then may happen that the solution changes significantly when the initial data change small. However, this phenomenon causes some difficulties on analysis of physical models and differential equations. Hence, it is often required the well-posedness of the Cauchy problem, which ensures that the solution continuously depends on initial data.

One of the distinguishing features of linear dispersive equations is that the speed of the spread of solutions depends on its frequency. Therefore, some kind of smoothing effect can be obtained by incorporating the oscillation caused by dispersion due to time evolution into the analysis. Furthermore, by grasping what kind of oscillation is generated from each nonlinear term, we can use the smoothing effect in the analysis of nonlinear dispersive equations.

I am studying the well-posedness of the Cauchy problem for dispersive equations by using the smoothing effect due to such a dispersive nature. Recently, I am also interested in asymptotic behavior of solutions to the Cauchy problem and stochastic dispersive equations.

Shinnosuke OKAWA

Algebraic Geometry

Geometric objects which are described as the collection of solutions of algebraic equation(s), such as ellipses, parabolas, and hyperbolas, are called algebraic varieties. These are the subject of study in the field of algebraic geometry, and I have been working on various problems in this area.

In the early days, I was studying topics about Geometric Invariant theory (GIT) and birational geometry. GIT is a theory about quotients of algebraic varieties by algebraic group actions, and in birational geometry certain "transformation (modification)" of algebraic varieties is studied. The definition of a GIT quotient depends on a choice of a parameter, which is called a stability condition, and one obtains different quotients by changing the stability conditions. Typically these quotients are all birational to each other, and in good situations it turns out that the birational geometry of the quotient variety is completely described in this way. I have proved several properties of this class of varieties.

Recently I am mainly investigating algebraic varieties from categorical points of view. One of my interests is the "irreducible decomposition" of the derived category of coherent sheaves. This in fact is motivated by birational geometry, and important techniques of birational geometry, e.g. the canonical bundle formula, are used. I am also studying non-commutative deformations of algebraic varieties and their moduli spaces. Derived categories again play a central role, but other interesting topics such as GIT, geometry of elliptic curves, and birational transformation of non-commutative algebraic varieties also show up. Through the study, I found an interesting and unexpected relationship with an old invariant theory which goes back to the end of the 19th century. I am also trying to understand to what extent the derived category of coherent sheaves keeps the geometric information of the original variety.

Yuichi SHIOZAWA

Probability Theory

My research area is probability theory. In particular, I am working on the sample path analysis for symmetric Markov processes generated by Dirichlet forms. Dirichlet form is defined as a closed Markovian bilinear form on the space of square integrable functions. The theory of Dirichlet forms plays important roles in order to construct and analyze symmetric Markov processes.

I am interested in the relation between the analytic information on Dirichlet forms and the sample path properties of symmetric Markov processes. I am also interested in the global properties of branching Markov processes, which are a mathematical model for the population growth of particles by branching.



Hiroshi SUGITA

Probability Theory

I specialize in Probability theory. In particular, I am interested in infinite dimensional stochastic analysis, Monte-Carlo method, and probabilistic number theory. Here I write about the Monte-Carlo method. One of the advanced features of the modern probability theory is that it can deal with "infinite number of random variables". It was E. Borel who first formulated "infinite number of coin tosses" on the Lebesgue probability space, i.e., a probability space consisting of $[0,1)$ -interval and the Lebesgue measure. It is a remarkable fact that all of useful objects in probability theory can be constructed upon these "infinite number of coin tosses". This fact is essential in the Monte-Carlo method. Indeed, in the Monte-Carlo method, we first construct our target random variable S as a function of coin tosses. Then we compute a sample of S by plugging a sample sequence of

coin tosses ω , which is computed by a pseudo-random generator, ω into the function. Now, a serious problem arises: How do we realize a pseudo-random generator? Can we find a perfect pseudo-random generator? People have believed it to be impossible for a long time. But in 1980s, a new notion of "computationally secure pseudo-random generator" let people believe that an imperfect pseudo-random generator has some possibility to be useful for practical purposes. Several years ago, I constructed and implemented a perfect pseudo-random generator for Monte-Carlo integration, i.e., one of Monte-Carlo methods which computes the mean values of random variables by utilizing the law of large numbers.

Atsushi TAKAHASHI

Complex Geometry, Algebra, Mathematical Physics

My current interests are mathematical aspects of the superstring theory, in particular, algebraic geometry related to the mirror symmetry.

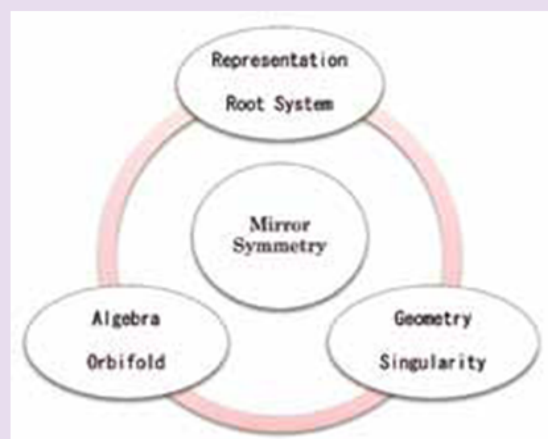
More precisely, I am studying homological algebras and moduli problems for categories of "D-branes" that extend derived categories of coherent sheaves on algebraic varieties.

Indeed, I am trying to construct Kyoji Saito's primitive forms and their associated Frobenius structures from triangulated categories defined via matrix factorizations attached to weighted homogeneous polynomials.

For example, I proved that the triangulated category for a polynomial of type ADE is equivalent to the derived category of finitely generated modules over the path algebra of the Dynkin quiver of the same type.

Now, I extend this result to the case when the polynomial corresponds to one of Arnold's 14

exceptional singularities and then showed the "mirror symmetry" between weighted homogeneous singularities and finite dimensional algebras, where a natural interpretation of the "Arnold's strange duality" is given.



Naohito TOMITA

Real Analysis

My research field is Fourier analysis, and I am particularly interested in the theory of function spaces. Fourier series were introduced by J. Fourier(1768-1830) for the purpose of solving the heat equation. Fourier considered as follows:"Trigonometric series can represent arbitrary periodic functions". However, in general, this is not true. Then, we have the following problem: "When can we write a periodic function as an infinite (or finite) sum of sine and cosine functions?". Lebesgue space which is one of function spaces plays an important role in this classical problem. Here Lebesgue space consists of functions whose p-th powers are integrable. In this way, function spaces are useful for various mathematical problems. As another

example, modulation spaces were recently applied to pseudodifferential operators which are important tool for partial differential equations, and my purpose is to clarify their relation.

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Kenkichi TSUNODA

Probability Theory

My research field is probability theory. In particular I am interested in problems related to so-called "Hydrodynamic limit", which is a certain type of space-time scaling limits. Hydrodynamic limit means a method which determines a macroscopic quantity of a microscopic system such as particles systems. To tackle a difficult problem related to Hydrodynamic limit, it is necessary to invoke results on functional analysis or partial differential equations, and to use specific arguments for particle systems and wide knowledge of probability theory. Hydrodynamic limit is formulated as Law of large numbers for a macroscopic quantity such as the number of particle systems or the current for a microscopic system. I am

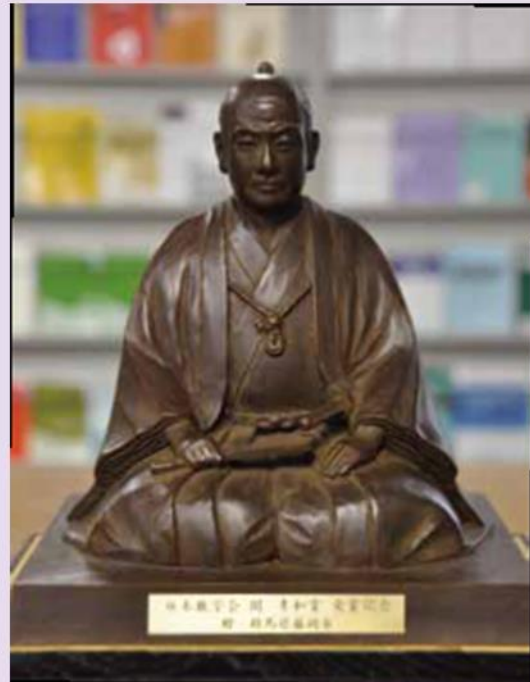
working on related Central limit theorem and Large deviation principle.

In recent years, my another interest is Random topology, which has arisen from the development of Topological data analysis in applied mathematics. I am working on this new research area with my probabilistic technique although this theme is not related to above Hydrodynamic limit deeply.

Motoo UCHIDA

Algebraic Analysis, Microlocal Analysis

My research field is algebraic analysis and micro-local analysis of partial differential equations. The view point of micro-local analysis (with cohomology) is a new important point of view in analysis introduced by Mikio Sato in the early 1970s. Thinking from a micro-local point of view helps us to well understand a number of mathematical phenomena (at least for PDE) and to find a simple hidden principle behind them. Even for some classical facts (scattered as well known results) we can sometimes find a new unified way of understanding from a micro-local or algebro-analytic viewpoint.



Takao WATANABE

Algebraic Number Theory

My current interest is the Geometry of Numbers. The Geometry of Numbers was founded by Hermann Minkowski in the beginning of the 20th century. Minkowski proved a famous theorem known as "Minkowski's convex body theorem", which asserts that "there exists a non-zero integer point in V if V is an o -symmetric convex body in the n -dimensional Euclidean space whose volume is greater than 2^n ". When V is an ellipsoid, this theorem is refined as follows. Let A be a non-singular 3 by 3 real matrix and $K(c)$ the ellipsoid consisting of points x such that the inner product (Ax, Ax) is less than or equal to $c > 0$. For $i = 1, 2, 3$, we define the constant c_i as the minimum of $c > 0$ such that $K(c)$ contains i linearly independent integer points. Then c_1, c_2, c_3 satisfies the inequality $c_1 c_2 c_3 \leq 2 |\det A|^2$. This is called "Minkowski's second theorem". A similar inequality holds for any n -dimensional ellipsoid. Namely, if A is a non-singular n by n real matrix and $K(c)$ is the n -dimensional ellipsoid defined by $(Ax, Ax) \leq c$, we can define c_i for $i = 1, 2, \dots, n$ as the

minimum of $c > 0$ such that $K(c)$ contains i linearly independent integer points. Then the inequality $c_1 c_2 \dots c_n \leq h(n) |\det A|^2$ holds for any A . The optimal upper bound $h(n)$ does not depend on A , and is called Hermite's constant. We know $h(2) = 4/3$, $h(3) = 2$, $h(4) = 4$, ..., $h(8) = 256$, but $h(n)$ for a general n is not known. A recent major topic of this research area is the determination of $h(24)$. In 2003, Henry Cohn and Abhinav Kumar proved that $h(24) = 4^{24}$. (Incidentally, $h(3)$ was essentially determined by Gauss in 1831, and $h(8)$ was determined by Blichfeldt in 1953. If you would determine $h(9)$, then your name would be recorded in treatises on the Geometry of Numbers.) Now I study (an analogue of) the Geometry of Numbers on algebraic homogeneous spaces. One of my results is a generalization of Minkowski's second theorem to a Severi-Berauer variety. In addition, I am interested in the reduction theory of arithmetic subgroups, automorphic forms, the algebraic theory of quadratic forms and Diophantine approximation.

Katsutoshi YAMANOI

Complex Analysis, Complex Geometry

My research interest is Complex geometry and Complex analysis, both from the view point of Nevanlinna theory. In the geometric side, I am interested in the conjectural second main theorem in the higher dimensional Nevanlinna theory for entire holomorphic curves into projective manifolds. Also I am interested in the behavior of Kobayashi pseudo-distance of projective manifolds of general type. These problems are related to an algebraic geometric problem of bounding the canonical degree of algebraic curves in projective manifolds of general type by the geometric genus of the curves. In the analytic side, I am interested in classical problems of value distribution theory for meromorphic functions in the complex plane.



Takehiko Yasuda

Algebraic Geometry, Singularity theory

My main research object is singularities of algebraic varieties. An algebraic variety is a "figure" formed by solutions of algebraic equations. Such a figure often has points where the figure is sharp-pointed or intersects itself. Singularities make the study of an algebraic variety difficult. However since they often appear under various constructions, it is important to study them. Also singularities are interesting research object themselves. More specifically, I am interested in resolution of singularities, the birational-geometric aspect of singularities, the McKay correspondence.

Although these are classical research areas, changing a viewpoint or the setting of a problem, one can sometimes find new phenomenon. Such a discovery is the greatest pleasure in my mathematical research. To pursue research, I use various tools like motivic integration, Frobenius maps, moduli-theoretic blowups, non-commutative rings, and sometimes make ones by myself. Recently I am fascinated by mysterious behaviors of singularities in positive characteristic (a world where summing up several 1's gives 0.)

SEMINARS and COLLOQUIA

● ALGEBRA

Department
of
Mathematics

Number Theory Seminar

Number theory seminar at Osaka University is a seminar for faculty members and graduate students of Osaka University or researchers studying nearby Osaka University. The seminar is usually held on Fridays, once every two weeks. The subject of the seminar covers wide topics concerning Number theory, especially, algebraic number theory and analytic number theory, modular forms, arithmetic geometry, representation theory and algebraic combinatorics. In this seminar, we have reports of new results on these topics and we exchange ideas and technics of our research.

Department
of
Mathematics

Algebraic Geometry Seminar

The seminar is held two or three times a month and each time one speaker gives a talk of 90 minutes. After a talk, we have time for questions and discussion. The purpose of the seminar is to learn important results by active researchers in Algebraic Geometry and related fields, providing new perspectives on the areas through lectures and discussions. We also have survey lectures by experts for graduate students and young researchers. We have guest speakers not only from domestic universities but also from foreign countries, reflecting various aspects of the research area.

● GEOMETRY

Department
of
Mathematics

Geometry Seminar

This seminar on Mondays is intended for talks that will be of interest to a wide range of geometers. Topics discussed include Riemannian, complex, and symplectic geometry; PDEs on manifolds; mathematical physics.

Department
of
Mathematics

Topology Seminar

In our research group of topology, we hold three kinds of specialised seminars regularly: the low-dimensional topology seminar focusing on the knot theory, three-manifolds, and hyperbolic geometry; the seminar on transformation groups; and the seminar on 4-manifolds and complex surfaces.

We also sometimes hold a topology seminar encompassing all fields of topology, where all of us meet together.

● ANALYSIS

Department
of
Mathematics

Seminar of Differential Equations

Our seminar is held on every Friday from 15:30 to 17:00. One of the features of the seminar is to cover a wide variety of topics on Qualitative Analysis of Differential Equations. In fact, we are interested in ordinary differential equations, partial differential equations, linear differential equations, nonlinear differential equations and so on. Lecturers are invited from not only domestic universities but also foreign countries and present us their original results or survey of recent development of their fields. Furthermore, this seminar provides opportunities to give a talk for our colleagues and Ph.D. students majoring in differential equations. Moreover, we should mention that we are pleased to have participants from other universities located closed to ours. In this way, we communicate with each other and try to contribute to the progress of the theory of differential equations.

Department
of
Mathematics

Seminar on Probability

Probability theory group, the graduate school of science and the graduate school of engineering science, organizes "Seminar on Probability" on Tuesday evening. The topics on this seminar are the following:

(1) Probability theory

Stochastic analysis and infinite dimensional analysis, problems arising from other areas of mathematics such as real analysis, differential equations and differential geometry.

(2) Research fields related to Probability theory, Ergodic theory, dynamical system, stochastic control and mathematical finance.

We welcome visits and talks by many researchers from other universities, domestic and abroad.

Department
of
Mathematics

Dynamics and Fractals Seminar

Researchers and students working on various fields related to dynamical systems and fractals attend this seminar. We meet once a month for approximately 90 minutes. Each talk on his/her research is followed by discussions among all participants.

Department
of
Mathematics

Mathematics Colloquium

Colloquia take place on Monday afternoon at 16 : 30 in Room E404. They are directed toward a general mathematical audience. In particular, one of the functions of these Colloquia is to inform non-specialists and graduate students about recent trends, ideas and results in some area of mathematics, or closely related fields.

● COLLOQUIA