Department of Mathematics

Summary

Mathematics, originally centered in the concepts of number, magnitude, and form, has long been growing since ancient Egyptian times to the 21st century. Through the use of abstraction and logical reasoning, it became an indispensable tool not only in natural sciences, but also in engineering and social sciences. Recently, the remarkable development of computers is now making an epoch in the history of mathematics.

Department of Mathematics is one of the six departments of Graduate School of Science, Osaka University. It consists of 6 research groups, all of which are actively involved in the latest developments of mathematics. Our mathematics department has ranked among the top seven in the country.

The department offers a program with 32 new students enrolled annually leading to post-graduate degrees of Masters of Science. The department also offers a Ph.D. program with possibly 16 new students enrolled annually.

Graduate courses are prepared so as to meet various demands of students. Besides introductory courses for first year students, a number of topics courses are given for advanced students. Students learn more specialized topics from seminars under the guidance of thesis advisors.

Our department has our own library equipped with about 500 academic journals and 50,000 books in mathematics, both of which graduate students can use freely. Also by an online system, students as well as faculty members can look up references through Internet.

Research Groups

Algebra, Geometry, Analysis, Global Geometry & Analysis, Experimental Mathematics, Mathematical Science.

Areas of Research

Number Theory, Commutative Ring Theory, Algebraic Geometry, Algebraic Analysis, Partial Differential Equations, Differential Geometry, Complex Differential Geometry, Topology, Knot Theory, Discrete Subgroups, Transformation Groups, Complex Analysis, Complex Functions of Several Variables, Complex Manifolds, Discrete Mathematics, Spectral Theory, Dynamical Systems, Fractals, Mathematical Engineering, Information Geometry, Information Theory, Cryptography.

Faculty Members

Professors (15)

Shin-ichi DOI, Osamu FUJINO, Akio FUJIWARA, Ryushi GOTO, Nakao HAYASHI, Soichiro KATAYAMA, Kazuhiro KONNO, Takehiko MORITA, Hiroaki NAKAMURA, Ken'ichi OHSHIKA, Shin-ichi OHTA, Hiroshi SUGITA, Atsushi TAKAHASHI, Takao WATANABE, Katsutoshi YAMANOI.

Associate Professors (15)

Ichiro ENOKI, Tetsuya ITO, Hisashi KASUYA, Eiko KIN, Hideki MIYACHI, Haruya MIZUTANI, Tomonori MORIYAMA, Tadashi OCHIAI, Shinnosuke OKAWA, Yuichi SHIOZAWA, Hideaki SUNAGAWA, Naohito TOMITA, Motoo UCHIDA, Seidai YASUDA, Takehiko YASUDA.

Lecturer (1)

Kazunori KIKUCHI.

Assistant Professors (6)

Yasuhiro HARA, Takao IOHARA, Ryo KANDA, Yoshihiko MATSUMOTO, Hiroyuki OGAWA, Koji OHNO.

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Professors (6)

Susumu ARIKI, Takayuki HIBI, Katsuhisa MIMACHI, Kenji NAKANISHI, Katsuhiro UNO, Masaaki WADA.

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Home Page

http://www.math.sci.osaka-u.ac.jp/eng/

Department of Mathematics

Shin-ichi DOI

Partial Differential Equations

Partial differential equations have their origins in various fields such as mathematical physics, differential geometry, and technology. Among them I am particularly interested in the partial differential equations that describe wave propagation phenomena: hyperbolic equations and dispersive equations. A typical example of the former is the wave equation, and that of the latter is the Schroedinger equation. For many years I have studied basic problems for these equations: existence and uniqueness of solutions, structure of singularities of solutions, asymptotic behavior of solutions, and spectral properties. Recently I make efforts to understand how the singularities of solutions for Schroedinger equations or, more generally, dispersive equations propagate. The center of this problem is to determine when and how the singularities of solutions for the dispersive equations can be described by the asymptotic behavior of solutions for the associated canonical equations.

Department of Mathematics

Ichiro ENOKI

Complex Differential Geometry

A complex manifold is, locally, the world build out of open subsets of complex Euclid spaces and holomorphic functions on them. If two holomorphic functions are defined on a connected set and coincide on an open subset, then they coincide on the whole. Complex manifolds inherit this kind of property from holomorphic functions. That is, they are stiff and hard in a sense. It seems to me that complex manifolds are not metallically hard but have common warm feeling with wood or bamboo, which have grain and gnarl. Analytic continuations, as you learned in the course on the function theory of complex variables, is analogous to the process of growth of plants. Instead of considering whole holomorphic functions, a class of complex manifold can be build out of polynomials. This is the world of complex algebraic manifolds, the most fertile area in the world of complex manifolds. To complex algebraic manifolds, since they are algebraically defined, algebraic methods are of course useful to study them. In certain cases, however, transcendental methods (the word "transcendental" means only "not algebraic") are powerful. For example, one of the simplest proof for the fundamental theorem of algebra is given by the function theory of one complex variable. These two methods have been competing with each other since the very beginning of the history of the study of complex manifolds. This competing seems to me the prime mover of the development of the theory of complex manifolds. Comparing the world of complex manifold to the earth, the world of complex algebraic manifold is to compare to continents, and the boundary to continental shells. The reason I wanted to begin to study complex manifolds was I heard the Kodaira embedding theorem, which characterizes complex algebraic manifolds in the whole complex manifolds. The place I begin to study is, however, something like the North Pole or the Mariana Trench. Now the center of my interest is in the study of complex algebraic manifolds by transcendental methods. (Thus I have reached land but I found this was a jungle.)

Department of Mathematics

Osamu FUJINO

Algebraic Geometry

I am mainly interested in algebraic geometry. More precisely, I am working on the birational geometry of higher-dimensional algebraic varieties. In the early 1980s, Shigefumi Mori initiated a new approach for higher-dimensional birational geometry, which is now usually called the Minimal Model Program or Mori theory. Unfortunately, this beautiful approach has not been completed yet. One of my dreams is to complete the Minimal Model Program in full generality. I am also interested in toric geometry, Hodge theory, complex geometry, and so on.

Department of Mathematics

Akio FUJIWARA

Mathematical Engineering

"What is information?" Having this naive yet profound question in mind, I have been studying mainly on quantum information theory, noncommutative statistics, information geometry, and algorithmic randomness theory. Quantum information theory is a quantum extension of classical information theory. Since Shor's invention of a factorizing algorithm on a quantum computer, it is one of the most exciting research field in information science. Among diverse branches of research subjects, I am interested in quantum channel coding theory, especially in the additivity problem of the quantum channel capacity. One may conceive of quantum theory as a noncommutative extension of probability theory. Likewise, noncommutative statistics is a quantum extension of classical statistics. It aims to find the optimal strategy for identifying an unknown quantum object from a statistical point of view. My interest includes quantum state/channel estimation, and quantum hypothesis testing. Probability theory is usually regarded as a branch of analysis. Yet it is also possible to investigate the space of probability measures from a differential geometrical point of view. Information geometry deals with a pair of affine connections which are mutually dual (conjugate) with respect to a Riemannian metric on a statistical manifold. It is well known that geometrical methods provide a useful guiding principle as well as insightful intuition in classical statistics. I am extending such a geometrical structure to a quantum regime, not just formally but admitting a fruitful operational interpretation. Finally, I am now delving into algorithmic randomness theory from an information geometrical point of view. My dream is to reformulate thermal/statistical physics in terms of algorithmic information theory.

Department of Mathematics

Ryushi GOTO

Geometry

My research interest is mostly in complex and differential geometry, which are closely related with algebraic geometry and theoretical physics. My own research started with special geometric structures such as Calabi-Yau, hyperKaehler, G2 and Spin(7) structures. These four structures exactly correspond to special holonomy groups which give rise to Ricci-flat Einstein metrics on manifolds. It is intriguing that these moduli spaces are smooth manifolds on which local Torelli type theorem holds. In order to understand these phenomena, I introduce a notion of geometric structures defined by a system of closed differential forms and establish a criterion of unobstructed deformations of structures. When we apply this approach to Calabi-Yau, hyperKaehler, G2 and Spin(7) structures, we obtain a unified construction of these moduli spaces. At present I also explore other interesting geometric structures and their moduli spaces.

Department of Mathematics

Yasuhiro HARA

Topology

The field of my study is topology and, especially, I study the theory of transformation groups. The Borsuk-Ulam theorem is one of famous theorems about transformation groups. This theorem is often taken up as an application in elementary lectures about the homology theory. The content of the theorem is as follows: for every continuous map from the n-dimensional sphere to the n-dimensional Euclidian space, there exists a point such that the map takes the same value at the point and at the antipodal point. A famous application of this theorem is the following. "On the earth, there is a point such that the temperature and humidity at the point are the same as those at the antipodal point." We consider a free action of a group of order two on the n-dimensional sphere to prove the Borsuk-Ulam theorem. Then for any equivariant map (any continuous map which preserves the structure of the group action) from the sphere to itself, the degree of the map is odd. By using this fact, we obtain the Borsuk-Ulam theorem. In the case of the Borsuk-Ulam theorem, we consider spheres and free actions of a group of order two. Actually, when we consider other manifolds and actions of other groups, there are some restrictions of homotopy types of equivariant maps. I study such restrictions of homotopy types of equivariant maps by using the cohomology theory, and I study relationships between homotopy types of equivariant maps and topological invariants.

Department of Mathematics

Nakao HAYASHI

Partial Differential Equations

I am interested in asymptotic behavior in time of solutions to nonlinear dispersive equations (1 D nonlinear Schroedinger, Benjamin-Ono, Korteweg-de Vries, modified Korteweg-de Vries, derivative nonlinear Schroedinger equations) and nonlinear dissipative equations Complex Landau-Ginzburg equations, Korteweg-de Vries equation on a half line, Damped wave equations with a critical nonlinearity). These equations have important physical applications. Exact solutions of the cubic nonlinear Schroedinger equations and Korteweg-de Vries can be obtained by using the inverse scattering method. Our aim is to study asymptotic properties of these nonlinear equations with general setting through the functional analysis. We also study nonlinear Schroedinger equations in general space dimensions with a critical nonlinearity of order 1+2/n and the Hartree equation, which is considered as a critical case and the inverse scattering method does not work. On 1995, Pavel I. Naumkin and I started to study the large time behavior of small solutions of the initial value problem for the non-linear dispersive equations and we obtained asymptotic behavior in time of solutions and existence of modified scattering states to nonlinear Schroedinger with critical and subcritical nonlinearities. It is known that the usual scattering states in L^2 do not exist in these equations. Recently, E.I.Kaikina and I are studying nonlinear dissipative equations (including Korteweg-de Vries) on a half line and some results concerning asymptotic behavior in time of solutions are obtained.

Department of Mathematics

Takao IOHARA

Nonlinear Partial Differential Equations

My research interest is concerned with nonlinear partial differential equations appearing in fluid mechanics. The current research topic is the equations of the motion of viscous incompressible fluid which has free moving surface. The motion of viscous incompressible fluid is governed by the Navier-Stokes equations, which are not easy to solve because of their nonlinearity. The free moving surface adds another nonlinearity to the problem and the study of it needs more elaborate technique than the problems on fixed domain.

Department of Mathematics

Tetsuya ITO

Topology, Combinatorial and Geometric Group Theory

I am studying topology, geometry and combinatorics concerning the braid groups, orderable groups and 3-manifolds. The braid group is illustrated by strands in 3-space and is intuitively easy to understand, but it plays an important role in many branch of mathematics, like quantum topology or contact geometry.

An orderable group is a group having a total ordering that is invariant under the multiplication of the group itself. It is also an interesting object related to one-dimensional dynamics and various other fields. I am mainly interested in an application of orderings to topology, and studying an explicit construction of orderings having strange properties such as isolated orderings.

Recently I am studying topology and contact structure of 3-manifolds via open book decomposition, which can be seen as a generalization of closed braids.

Department of Mathematics

Ryo KANDA

Ring Theory

My research area is ring theory. A ring is an algebraic structure which has addition, subtraction, and multiplication. Typical examples are the set of integers, the set of polynomials, and the set of n-by-n matrices. I am particularly interested in noncommutative rings, whose multiplication is noncommutative.

The notion of modules plays an important role in the study of a ring. It is a generalization of vector spaces appearing in linear algebra, but the only difference in the definition is that the coefficients live in a ring, not necessarily a field such as the field of real/complex numbers. A vector space over an arbitrary coefficient field is determined by the cardinality of its basis, up to isomorphism. On the other hand, in the case of a ring, even the nature of finitely generated modules highly depends on the structure of the ring. For this reason, we can investigate the ring by looking the behavior of its modules. Especially for noncommutative rings, this approach is often clearer than looking the ring itself directly, and this leads us to deeper results.

An abelian category is a further generalization of the collection of modules. For each ring, the collection of its modules has the structure of an abelian category, and the ring can be almost recovered by the categorical structure. A similar thing holds for the abelian category consisting of coherent sheaves on an algebraic variety. Hence the notion of abelian categories is a large framework including (noncommutative) rings and (commutative) algebraic varieties. I have investigated general properties of certain classes of abelian categories, and have revealed several new properties of noncommutative rings as consequences. An advantage of this general setting is that we can consider similar problems for abelian categories which are not obtained as module categories over rings. For example, the functor category, which is the category consisting of functors from a given abelian category, is again an abelian category, and its structure reflects homological properties of the original abelian category. I expect that, by considering naive questions arising from general theory of abelian categories in a specific setting, such as the functor category, we can extend existing theories to new directions.

Department of Mathematics

Hisashi KASUYA

Geometry

Until now, I have tried to extend the geometry of nilpotent groups to the geometry of solvable groups. More precisely, I have studied the cohomology theory of homogeneous spaces of solvable Lie groups and complex geometry of non-Kahler manifolds. It seems that the gap between nilpotent groups and solvable groups is small. But this gap contains a potential for geometry. By the growing out of left-invariance and non-triviality of local system cohomology, I succeeded in giving a great surprise.

Recently, I am interested in the geometry which relates to reductive or semi-simple groups in contrast to nilpotent or solvable groups In particular, I study non-abelian Hodge theory, variations of hodge structures, lattices in semi-simple Lie groups and locally homogeneous spaces.

Department of Mathematics

Soichiro KATAYAMA

Nonlinear Partial Differential Equations

My research interest is in nonlinear partial differential equations. To be more specific, I am working on the initial value problem for nonlinear wave equations (in a narrow sense), and also for partial differential equations describing the nonlinear wave propagation in a wider sense, such as Klein-Gordon equations and Schroedinger equations.

The initial value problem is to find a solution to a given partial differential equation with a given state at the initial time (a given initial value). However, in general, it is almost impossible to give explicit expression of solutions to nonlinear equations. Therefore, in the mathematical theory, it is important to investigate the existence of solutions and also their behavior when they exist.

If we consider the initial value problem for the equations mentioned above and if the initial value is sufficiently small, the existence of solutions up to arbitrary time (the existence of global solutions) is mainly determined by the power of the nonlinearity. Especially, when the nonlinearity has the critical power, the existence and non-existence of global solutions depend also on the detailed structure of the nonlinear terms. I am interested in this kind of critical case, and studying sufficient conditions for the existence of global solutions and their asymptotic behavior.

Department of Mathematics

Kazunori KIKUCHI

Differential Topology

I have been studying topology of smooth four-dimensional manifolds, in particular interested in homology genera, representations of diffeomorphism groups to intersection forms, and branched coverings. Let me give a simple explanation of what interests me the most, or homology genera. The homology genus of a smooth four-dimensional manifold M is a map associating to each two-dimensional integral homology class [x] of M the minimal genus g of smooth surfaces in M that represent [x]. For simplicity, reducing the dimensions of M and [x] to the halves of them respectively, consider as a two-dimensional manifold the surface of a doughnut, or torus T, and a one-dimensional integral homology class [y] of T. Draw a meridian and a longitude on T as on the terrestrial sphere, and let [m] and [l] denote the homology classes of T represented by the meridian and the longitude respectively. It turns out that [y] = a[m] + b[l] for some integers a and b, and that [y] is represented by a circle immersed on T with only double points. Naturally interesting then is the following question: what is the minimal number n of the double points of such immersions representing [y]? Easy experiments would tell you that, for example, n = 0 when (a,b) = (1,0) or (0,1) and n = 1 when (a,b) = (2,0) or (0,2). In fact, it is proved with topological methods that n = d – 1, where d is the greatest common divisor of a and b. It is the minimal number n for T and [y] that corresponds to the minimal genus g for M and [x]. The study on the minimal genus g does not seem to proceed with only topological methods; it sometimes requires methods from differential geometry, in particular methods with gauge theory from physics; though more difficult, it is more interesting to me. I have been tackling the problem on the minimal genus g with such a topological way of thinking as to see things as if they were visible even though invisible.

Department of Mathematics

Eiko KIN

Geometric Topology

I am interested in the mapping class groups on surfaces. The most common elements in the mapping class groups are so called pseudo-Anosovs. I try to understand which pseudo-Anosovs are the most simplest one. Let me explain my goal more clearly. Pseudo-Anosovs possess many chaotic (and beautiful) properties from the view points of the dynamical systems and hyperbolic geometry. There are some quantities which reflect those complexities of pseudo-Anosovs. (Topological) entropies and (hyperbolic) volumes are examples. We fix a surface (say a closed surface of genus 5 which is my favorite one) and we consider the set of entropies (the set of volumes) coming from the pseudo-Anosov elements on the surface. Then one can see that there exists a minimum of the set. That is, we can talk about the pseudo-Anosovs with the minimal entropies (with the minimal volumes). I would like to know which pseudo-Anosov achieves the minimal entropy/ minimal volume. Recently, Gabai, Meyerhoff and Milley determined hyperbolic 3-manifolds with very small volume. Intriguingly, the result implies that those hyperbolic 3-manifolds are obtained from a single hyperbolic 3-manifold by Dehn filling. Some experts call such a single 3-manifold the "magic manifold". Said differently, the magic manifold is a parent manifold of the hyperbolic 3-manifolds with very small volume. It seems likely that we have the same story in the world of pseudo-Anosovs with the very small entropies. This conjecture is based on recent works of myself and other specialists. We note that there are infinitely many topological types of surfaces. (For example, a family of closed orientable surface of genus g). For the mapping class group of each surface, we know that there exists a pseudo-Anosov element with the minimal entropy. Thus, of course, there exist infinitely many pseudo-Anosov elements with the minimal entropies. It might be true that all minimizers are obtained from the magic 3-manifold. When I work on this project, I sometimes think of our universe.

Department of Mathematics

Kazuhiro KONNO

Complex Algebraic Geometry

Algebraic Geometry is a branch of Mathematics studying, by means of algebraic methods, the geometry of figures defined by simultaneous algebraic equations in several variables. You may say that you are not familiar with Algebraic Geometry. But you already know many beautiful plane curves such as an ellipse, a parabola and a hyperbola; they are in fact our jewels — algebraic varieties. As you learned in high school, various problems on the geometry of plane curves, e.g., how two curves intersect or contact, can be solved by considering simultaneous equations. Studying figures in such a way is nothing but the algebraic geometry. Algebraic equations, however, are not so simple; it is not known so far even how to solve simultaneous quadratic equations, whilst the method for linear equations are well established as you learned in the course of Linear Algebra, and many beautiful algebraic varieties are usually given by quadratic equations. Because in general we cannot draw figures of varieties on the black board, unlike ellipses or parabolas, it requires such and such training in order to be able to touch and feel them. For example, my favorite algebraic surfaces are 4 dimensional objects and, therefore, cannot be realized in our 3 dimensional space. If you are interested in meeting them in reality, the best way is to start and enjoy learning Algebraic Geometry.

Department of Mathematics

Yoshihiko MATSUMOTO

Differential Geometry, Several Complex Variables

Working on differential geometry, partly with some flavor of function theory of several complex variables. I've been mainly studying geometry of "asymptotically complex hyperbolic spaces," with emphasis on a partial differential equation called Einstein's equation on them. While being a generalization of geometry of bounded strictly pseudoconvex domains in function theory of several complex variables, it's beyond the scope of the field of complex geometry. Based on this experience, I'm now aiming toward some more general theory that applies to other "spaces that converge to ones with much symmetry," which are called "asymptotically symmetric spaces."

Asymptotically hyperbolic spaces, which are the most basic examples of asymptotically symmetric spaces, are lacking symmetries such as the homogeneity or the isotropicity of the genuine hyperbolic spaces in the strict sense. However, as a point in the space moves more away from a fixed one, its neighborhood looks more like an open set of the hyperbolic space. Recall the "natural" conformal structure on the sphere at infinity of the hyperbolic space—the boundary at infinity of an asymptotically hyperbolic space is equipped with a conformal structure in the same way. In the case of asymptotically complex hyperbolic spaces, whose model has little less isotropicity than that of the hyperbolic space, the associated geometric structure on the boundary at infinity is the CR (Cauchy–Riemann) structure.

The fundamental question of geometry of asymptotically symmetric spaces is the following: what property of the space reflects the geometric structure at infinity and the topology of the space, and in what way? This includes the question whether or not there is a space with some certain property under a given condition on the structure at infinity and the topology. The existence problem of Einstein metrics is a typical example.

Although asymptotically hyperbolic spaces have been studied for several decades, many fundamental questions remain unsolved. And, if we think of understandings from the general viewpoint of geometry of asymptotically symmetric spaces, our field still seems to be kind of a wilderness. I'd cultivate it by going back and forth between analyses on special cases and abstract considerations.

Department of Mathematics

Hideki MIYACHI

Hyperbolic Geometry , Teichmüller Theory

My research interests are in Teichmuller theory and geometric structures (especially Hyperbolic geometry) on manifolds, and their applications. I am recently interested in the mathematical phenomena which happen when geometric structures on surfaces (manifolds) are degenerating.

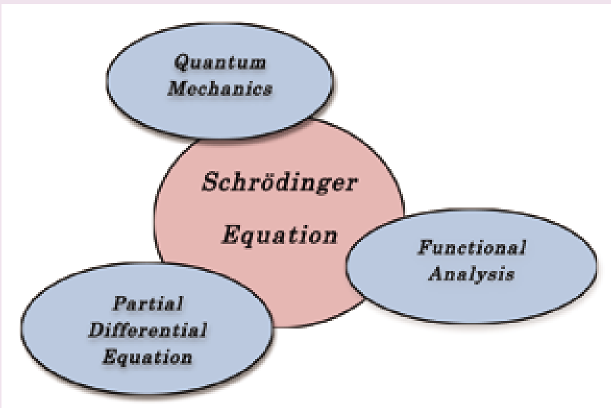
For instance, one of our problems is how the geometrical quantities behave under degenerations. Teichmüller theory is studied and applied in various fields including Complex analysis, Topology, Differential geometry and Algebraic geometry.

Department of Mathematics

Haruya MIZUTANI

Partial differential equations

The Schrödinger equation is the fundamental equation of physics for describing quantum mechanical behavior. I am working on the mathematical theory of the Schrödinger equation and my research interest includes scattering theory, semiclassical analysis, spectral theory, geometric microlocal analysis and so on. My current research has focused on various estimates such as decay or Strichartz inequalities, which describe dispersive or smoothing properties of solutions and are fundamental for studying linear and nonlinear dispersive equations. In particular, I am interested in understanding quantitatively the influence of the geometry of associated classical mechanics on the behavior of quantum mechanics, via such inequalities.



Department of Mathematics

Takehiko MORITA

Ergodic Theory

I specialize in ergodic theory. To be more precise, I am studying statistical behavior of dynamical systems via thermodynamic formalism and its applications.

Ergodic theory is a branch of mathematics that studies dynamical systems with measurable structure and related problems. Its origins can be found in the work of Boltzmann in the 1880s which is concerned with the so called Ergodic Hypothesis. Roughly speaking the hypothesis was introduced in order to guarantee that the system considered is ergodic i.e. the space averages and the long time averages of the physical observables coincide. Unfortunately, it turns out that dynamical systems are not always ergodic in general. Because of such a background, the ergodic problem (= the problem to determine a given dynamical system is ergodic or not) has been one of the important subjects since the theory came into existence. In nowadays ergodic theory has grown to be a huge branch and has applications not only to statistical mechanics, probability, and dynamical systems but also to number theory, differential geometry, functional analysis, and so on.

Department of Mathematics

Tomonori MORIYAMA

Number Theory

I am interested in automorphic forms of several variables. A classical automorpchic (modular) form of one variable is a holomorphic function on the upper half plane having certain symmetry. Such functions appear in various branches of mathematics, say notably number theory, and have been investigated by many mathematicians.

There is a family of minifolds called Riemannian symmetric spaces, which is a higher-dimensional generalization of the upper half plane. The set of isometries of a Riemannian symmetric space forms a Lie group G. Roughly speaking, an automorphic form of several variables is a function on a Riemannian symmetric space satisfying the relative invariance under an "arithmetic" subgroup of G and certain differential equations arising from the Lie group G. Studies on automorphic forms of several variables started from C. L. Siegel's works in 1930s and have been developed through interaction with mathematics of the day.

Currently I am working on two themes: (i) the zeta functions attached to automorphic forms and (ii) explicit constructions of automorphic forms, by employing representation theory of reductive groups over local fields. One of the joy in studying this area is to discover a surprisingly simple structure among seemingly complicated objects.

Department of Mathematics

Hiroaki NAKAMURA

Number Theory

Theory of equations has a long history of thousands of years in mathematics, and, passing publication of the famous Cardano-Ferrari formulas in Italian Renaissance, Galois theory in the 19th century established a necessary and sufficient condition for an algebraic equation to have a root solution in terms of its Galois group. My research interest is a modern version of Galois theory, especially its arithmetic aspects.

In the last century, the notion of Galois group was generalized to "arithmetic fundamental group" by Grothendieck, and Belyi's discovery (of an intimate relationship between Galois groups of algebraic numbers and fundamental groups of topological loops on hyperbolic curves) undertook a new area of "anabelian geometry". Here are important problems of controlling a series of covers of algebraic curves and their moduli spaces, and Ihara's theory found deep arithmetic phenomena therein.

Related also to Diophantus questions on rational points, fields of definitions and the inverse Galois problem, nowadays, there frequently occur important developments as well as new unsolved problems. I investigate these topics, and hope to find new perspectives for deeper understanding of the circle of ideas.

Department of Mathematics

Tadashi OCHIAI

Number theory, Arithmetic Geometry

I study Number theory and Arithmetic Geometry. In the research of Number theory, we study not only properties of integers and rational numbers, but every kinds of problems related to integers. For example, we are very much interested in rational points of algebraic varieties defined over the field of rational numbers. We have a long history of our research and we had a great progress on such problems through the proof of Fermat's conjecture by Wiles and the proof of Mordell's conjecture by Faltings in 20th century. My subject of research was first the study of geometry of arithmetic algebraic varieties using the l-adic etale cohomology of the variety and the Lefshetz trace formula. More recently, I am interested in the study of the special values of zeta functions via the philosophy of Iwasawa theory. My project is to study Iwasawa theory from the view point of Galois deformations. I think that Number theory is full of surprise as we see a lot of unexpected relations between different kinds of objects.

Department of Mathematics

Hiroyuki OGAWA

Number Theory

I have an interest in periodic objects. Expanding rational numbers into decimal numbers is delightful. The decimal number expansion becomes the repeat of a sequence of some integers. I have an appetite for continued fraction expansions, never get tired to calculate it, and want to find continued fractions with sufficiently long period. It is on the way to Gauss' class number one conjecture. Recently, I am studying iteration of rational functions. For a rational function g(x) with rational coefficients, a complex number z with g(g(...g(z)...))=z is called a periodic point on g(x) and is an algebraic number. I expect that number theoretical properties which such an algebraic number z has is described by the rational function g(x). This does not seem to work out anytime, but one can find many rational functions g(x) that describe the Galois group, the class number, the class group, and so on of a periodic point of g(x). I think that this should be surely useful, and calculate like these every day.

Department of Mathematics

Koji OHNO

Algebraic Geometry

When I was a student, I thought I knew number theory, geometry, but algebraic geometry was unfamiliar for me. One may say algebra and geometry are different fields, but you know the theory of quadratic curves and are aware of efficiency of algebraic methods for solving geometric problems. The field called algebraic geometry lies on such a line. When I was studying the theory of quadratic curves, I wondered, "Why do they only treat special equations like quadratics? There are many other equations. But how can they be treated?". When I discovered the answer might lie on this field, I decided to enter this field. The easiest non-trivial equation has the form such as "the second power of y = an equation of x of degree three", which defines the so called "an elliptic curve". The theory of elliptic curve was one of the greatest achievements of nineteenth century and keeps developing today. Recently, the famous Fermat's conjecture has been solved using this theory. The theory of quadratic and elliptic curves involve only two variables x, y. It is natural to think of the equations with many variables. In fact the algebraic geometrists are expanding the theory, curves to surfaces and higher dimensional cases these days. Two dimensional version of elliptic curves are called K3 surfaces, which can be treated only using the theory of linear algebra(!) thanks to the Torelli's theorem. These days, the 3-dimensional versions, which is called Calabi-Yau threefolds are fascinating for algebraic geometrists like me. Somehow theoretical physicists are also interested in this field. To study Calabi-Yaus by specializing these to ones with a fiber structure (on which field, I'm now working) might be one method, but I have been thinking that a new theory is needed. These days, many intriguing new theories have appeared and one may find more!

Department of Mathematics

Ken'ichi OHSHIKA

Topology

My speciality is studying 3-manifolds, hyperbolic geometry and discrete groups. The study of 3-manfolds is originated from the work of Poincare, and has been pursued without interruption, being harnessed by a naNove desire to understand the world in which we are living. At the turn of the present century, there was a big event, which was an affirmative resolution of the Thurston's geometrisation conjecture by Perelman. Still there remains a long way to go until we can say that we have completely understood 3-manifolds, and many topologists are actively studying them.

With the geometrisation conjecture being solved, the most important class of 3-manifolds is that of hyperbolic manifolds. Study of hyperbolic 3-manifolds is closely related to that of Kleinian groups, which has been done within the framework of complex analysis for a long time. I have been working in this field for a few decades.

The techniques and tools which were obtained in the filed of hyperbolic geometry are now combined with geometric group theory, which was initiated by Gromov, and flourishing enormously. I am also interested in this field, and my target at the moment is to cultivate a new perspective combining my viewpoint of Kleinian group theory with geometric group theory.

Department of Mathematics

Shin-ichi OHTA

Differential Geometry

My research subject is geometry, especially differential geometry and geometric analysis related to analysis and probability theory. A keyword of my research is “curvature” which represents how the space is curved. As seen in the difference between the sums of interior angles of triangles in a plane and a sphere, the behavior of the curvature influences various properties of the space (the shapes of triangles, the volume growth of concentric balls, how heat propagates, the behavior of entropy, etc.). This powerful and versatile conception has been applied to Riemannian manifolds, metric spaces, Finsler manifolds, Banach spaces, as well as discrete objects such as graphs.

Department of Mathematics

Shinnosuke OKAWA

Algebraic Geometry

Geometric objects which are described as the collection of solutions of algebraic equation(s), such as elliptic curves, parabolas, and hyperbolas, are called algebraic varieties. These are the subject of study in the field of algebraic geometry, and I have been working on various problems in this area.

In the early days, I was studying topics about Geometric Invariant theory (GIT) and birational geometry. GIT is a theory about quotients of algebraic varieties by algebraic group actions, and in birational geometry certain "transformation (modification)" of algebraic varieties is studied. The definition of a GIT quotient depends on a choice of a parameter, which is called a stability condition, and one obtains different quotients by changing the stability conditions. Typically these quotients are all birational to each other, and in good situations it turns out that the birational geometry of the quotient variety is complete described in this way. I have proved several properties of this class of varieties.

I slightly change my point of view and recently I am mainly investigating algebraic varieties from categorical points of view. One of my interests is the "irreducible decomposition" of the derived category of coherent sheaves. This in fact is motivated by birational geometry, and important techniques of birational geometry, e.g. the canonical bundle formula, are used. I am also studying non-commutative deformations of algebraic varieties and their moduli spaces. Derived categories again plays a central role, but other interesting topics such as GIT, geometry of elliptic curves, and birational transformation of non-commutative algebraic varieties also show up. Through the study, I found an interesting and unexpected relationship with an old invariant theory which goes back to the end of the 19th century. I am also trying to understand to what extent the derived category of coherent sheaves keeps the geometric information of the original variety.

Department of Mathematics

Yuichi SHIOZAWA

Probability Theory

My research area is probability theory. In particular, I am working on the sample path analysis for symmetric Markov processes generated by Dirichlet forms. Dirichlet form is defined as a closed Markovian bilinear form on the space of square integrable functions. The theory of Dirichlet forms plays important roles in order to construct and analyze symmetric Markov processes.

I am interested in the relation between the analytic information on Dirichlet forms and the sample path properties of symmetric Markov processes. I am also interested in the global properties of branching Markov processes, which are a mathematical model for the population growth of particles by branching.

Department of Mathematics

Hiroshi SUGITA

Probability Theory

I specialized in Probability theory. In particular, I am interested in infinite dimensional stochastic analysis, Monte-Carlo method, and probabilistic number theory. Here I write about the Monte-Carlo method. One of the advanced features of the modern probability theory is that it can deal with "infinite number of random variables". It was E. Borel who first formulated "infinite number of coin tosses" on the Lebesgue probability space, i.e., a probability space consisting of [0,1)-interval and the Lebesgue measure.

It is a remarkable fact that all of useful objects in probability theory can be constructed upon these "infinite number of coin tosses".

This fact is essential in the Monte-Carlo method. Indeed, in the Monte-Carlo method, we first construct our target random variable S as a function of coin tosses.

Then we compute a sample of S by plugging a sample sequence of coin tosses — , which is computed by a pseudo-random generator, — into the function.

Now, a serious problem arises: How do we realize a pseudo-random generator?

Can we find a perfect pseudo-random generator? People have believed it to be impossible for a long time. But in 1980s, a new notion of "computationally secure pseudo-random generator" let people believe that an imperfect pseudo-random generator has some possibility to be useful for practical purposes. A few years ago, I constructed and implemented a perfect pseudo-random generator for Monte-Carlo integration, i.e., one of Monte-Carlo methods which computes the mean values of random variables by utilizing the law of large numbers.

Department of Mathematics

Hideaki SUNAGAWA

Partial Differential Equations

My research field is Partial Differential Equations of hyperbolic and dispersive type. They arise in mathematical physics as equations describing wave propagation, so there are a wealth of applications and plenty of problems to be studied. Of my special interest is the nonlinear interactions of hyperbolic waves. Since the analysis of nonlinear PDE is still a developing subject, there are few general conclusions about that. To put it another way, it means that there are possibilities for coming across wonderful phenomena which no one has ever seen before.

Department of Mathematics

Atsushi TAKAHASHI

Algebraic Geometry, Mathematical Physics

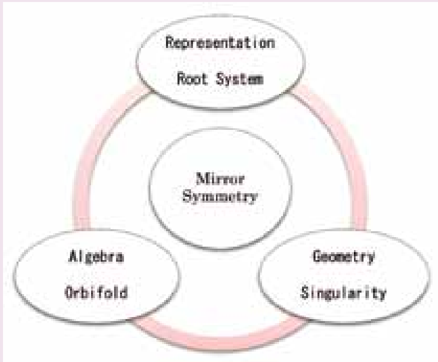
My current interests are mathematical aspects of the superstring theory, in particular, algebraic geometry related to the mirror symmetry.

More precisely, I am studying homological algebras and moduli problems for categories of "D-branes" that extend derived categories of coherent sheaves on algebraic varieties.

Indeed, I am trying to construct Kyoji Saito's primitive forms and their associated Frobenius structures from triangulated categories defined via matrix factorizations attached to weighted homogeneous polynomials.

For example, I proved that the triangulated category for a polynomial of type ADE is equivalent to the derived category of finitely generated modules over the path algebra of the Dynkin quiver of the same type.

Now, I extend this result to the case when the polynomial corresponds to one of Arnold's 14 exceptional singularities and then showed the "mirror symmetry" between weighted homogeneous singularities and finite dimensional algebras, where a natural interpretation of the "Arnold's strange duality" is given.



Department of Mathematics

Naohito TOMITA

Real Analysis

My research field is Fourier analysis, and I am particularly interested in the theory of function spaces. Fourier series were introduced by J. Fourier (1768-1830) for the purpose of solving the heat equation. Fourier considered as follows: "Trigonometric series can represent arbitrary periodic functions". However, in general, this is not true. Then, we have the following problem: "When can we write a periodic function as an infinite (or finite) sum of sine and cosine functions?". Lebesgue space which is one of function spaces plays an important role in this classical problem. Here Lebesgue space consists of functions whose p-th powers are integrable. In this way, function spaces are useful for various mathematical problems. As another example, modulation spaces were recently applied to pseudo-differential operators which are important tool for partial differential equations, and my purpose is to clarify their relation.

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$$

Department of Mathematics

Motoo UCHIDA

Algebraic Analysis

My research field is algebraic analysis and micro-local analysis of partial differential equations. The view point of micro-local analysis (with cohomology) is a new important point of view in analysis introduced by Mikio Sato in the early 1970s. Thinking from a micro-local point of view helps us to well understand a number of mathematical phenomena (at least for PDE) and to find a simple hidden principle behind them. Even for some classical facts (scattered as well known results) we can sometimes find a new unified way of understanding from a micro-local or algebro-analytic viewpoint.

Department of Mathematics

Takao WATANABE

Number Theory

My current interest is the Geometry of Numbers. The Geometry of Numbers was founded by Hermann Minkowski in the beginning of the 20th century. Minkowski proved a famous theorem known as "Minkowski's convex body theorem", which asserts that "there exists a non-zero integer point in V if V is an o-symmetric convex body in the n-dimensional Euclidean space whose volume is greater than 2^n". When V is an ellipsoid, this theorem is refined as follows. Let A be a non-singular 3 by 3 real matrix and K(c) the ellipsoid consisting of points x such that the inner product (Ax, Ax) is less than or equal to c > 0. For i = 1,2,3, we define the constant c_i as the minimum of c > 0 such that K(c) contains i linearly independent integer points. Then c_1, c_2, c_3 satisfies the inequality c_1c_2c_3 <= 2|det A|^2. This is called "Minkowski's second theorem". A similar inequality holds for any n-dimensional ellipsoid. Namely, if A is a non-singular n by n real matrix and K(c) is the n-dimensional ellipsoid defined by (Ax, Ax) <= c, we can define c_i for i = 1,2, ..., n as the minimum of c > 0 such that K(c) contains i linearly independent integer points. Then the inequality c_1c_2...c_n <= h(n)|det A|^2 holds for any A. The optimal upper bound h(n) does not depend on A, and is called Hermite's constant. We know h(2) = 4/3, h(3) = 2, h(4) = 4, ..., h(8) = 256, but h(n) for a general n is not known. A recent major topic of this research area is the determination of h(24). In 2003, Henry Cohn and Abhinav Kumar proved that h(24) = 4^24. (Incidentally, h(3) was essentially determined by Gauss in 1831, and h(8) was determined by Blichfeldt in 1953. If you would determine h(9), then your name would be recorded in treatises on the Geometry of Numbers.) Now I study (an analogue of) the Geometry of Numbers on algebraic homogeneous spaces. One of my results is a generalization of Minkowski's second theorem to a Severi-Berauer variety. In addition, I am interested in the reduction theory of arithmetic subgroups, automorphic forms, the algebraic theory of quadratic forms and Diophantine approximation.

Department of Mathematics

Katsutoshi YAMANOI

Complex Geometry

My research interest is Complex geometry and Complex analysis, both from the view point of Nevanlinna theory. In the geometric side, I am interested in the conjectural second main theorem in the higher dimensional Nevanlinna theory for entire holomorphic curves into projective manifolds. Also I am interested in the behavior of Kobayashi pseudo-distance of projective manifolds of general type. These problems are related to an algebraic geometric problem of bounding the canonical degree of algebraic curves in projective manifolds of general type by the geometric genus of the curves. In the analytic side, I am interested in classical problems of value distribution theory for meromorphic functions in the complex plane.

Department of Mathematics

Seidai YASUDA

Number Theory

I am interested in the special values of L-functions, especially Hasse-Weil L functions. Hasse-Weil L-functions are defined for arithmetic schemes or motives by using geometric cohomology. I also study multiple zeta values.

Systems of polynomials with integral coefficients are studied in number theory. It is often very difficult to find the integral solutions of such a system. Instead, we simultaneously deal with the solutions in various commutative rings. The solutions in various rings forms a scheme, which provides geometric methods for studying the system of polynomials.

Special values of Hasse-Weil L functions are believed to be related to motivic cohomologies, which are defined by using algebraic cycles or algebraic K-theory and are usually they hard to know explicitly. It is a very deep prediction which relates abstract objects with the concrete objects, L-functions. Multiple zeta values are related not only to the fundamental groups of moduli spaces of genus zero curves, mixed Tate motives, but also KZ equation, and Drinfeld associators.

It is expected that motives are related to automorphic representations. The expectation is important since we have various methods for studying automorphic L-functions. Some relations between motives and automorphic representations are realized by using Shimura varieties. In a joint work with Satoshi Kondo, I have proved a equality relating motivic cohomologies and special values of Hasse-Weil L-functions for some function field analogues of Shimura varieties.

Hasse-Weil L-functions are defined via some Galois representations. We need to study such Galois representations. For some technical reasons it is important to study Galois representations of p-adic fields with p-adic coefficients, and p-adic Hodge theory provides some tools for studying such representations. For recent years there have been much development in p-adic Hodge theory, and a lot of beautiful theories have been constructed. However the the theory is not fully established and many aspects of the theory remains mysterious. I am now trying to make the integral p-adic Hodge theory more convenient for explicit computation.

Department of Mathematics

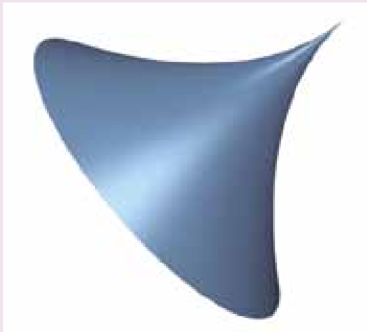
Takehiko YASUDA

Algebraic Geometry, Singularity Theory

My main research object is singularities of algebraic varieties. An algebraic variety is a "figure" formed by solutions of algebraic equations. Such a figure often has points where the figure is sharp-pointed or intersects itself. Singularities make the study of an algebraic variety difficult. However since they often appear under various constructions, it is important to study them. Also singularities are interesting research object themselves.

More specifically, I am interested in resolution of singularities, the birational-geometric aspect of singularities, the McKay correspondence. Although these are classical research areas, changing a viewpoint or the setting of a problem, one can sometimes find a new phenomenon. Such a discovery is the greatest pleasure in my mathematical research. To pursue research, I use various tools like motivic integration, Frobenius maps, moduli-theoretic blowups, non-commutative rings, and sometimes make ones by myself.

Recently I am fascinated by mysterious behaviors of singularities in positive characteristic (a world where summing up several 1's gives 0.)



SEMINARS and COLLOQUIA

●ALGEBRA

Department of Mathematics

Number Theory Seminar

Number theory seminar at Osaka University is a seminar for faculty members and graduate students of Osaka University or researchers studying nearby Osaka University. The seminar is usually held on Fridays, once every two weeks. The subject of the seminar covers wide topics concerning Number theory, especially, algebraic number theory, analytic number theory, modular forms, arithmetic geometry, representation theory and algebraic combinatorics. In this seminar, we have reports of new results on these topics and we exchange ideas and technics of our research.

Department of Mathematics

Algebraic Geometry Seminar

The seminar is held two or three times a month and each time one speaker gives a talk of 90 minutes. After a talk, we have time for questions and discussion. The purpose of the seminar is to learn important results by active researchers in Algebraic Geometry and related fields, providing new perspectives on the areas through lectures and discussions. We also have survey lectures by experts for graduate students and young researchers. We have guest speakers not only from domestic universities but also from foreign countries, reflecting various aspects of the research area.

●GEOMETRY

Department of Mathematics

Geometry Seminar

This seminar on Mondays is intended for talks that will be of interest to a wide range of geometers. Topics discussed include Riemannian, complex, and symplectic geometry; PDEs on manifolds; mathematical physics.

Department of Mathematics

Topology Seminar

In our research group of topology, we hold three kinds of specialised seminars regularly: the low-dimensional topology seminar focusing on the knot theory, three-manifolds, and hyperbolic geometry; the seminar on transformation groups; and the seminar on four-manifolds focused on 4-manifolds and complex surfaces.

We also sometimes hold a topology seminar encompassing all fields of topology, where all of us meet together.

●ANALYSIS

Department of Mathematics

Seminar of Differential Equations

Our seminar is held every Friday from 15:30 to 17:00. One of the features of the seminar is to cover a wide variety of topics on Qualitative Analysis of Differential Equations. In fact, we are interested in ordinary differential equations, partial differential equations, linear differential equations, nonlinear differential equations and so on. Lecturers are invited from not only domestic universities, but also foreign countries and present us their original results or survey of recent development of their fields. Furthermore, this seminar provides opportunities to give a talk for our colleagues and Ph.D. students majoring in differential equations. Moreover, we should mention that we are pleased to have participants from other universities located close to ours. In this way we communicate with each other and try to contribute to the progress of the theory of differential equations.

Department of Mathematics

Seminar on Probability

Probability theory group, the graduate school of science and the graduate school of engineering science, organizes "Seminar on Probability" on Tuesday evening.
The topics on this seminar are the following:

(1) Probability theory
Stochastic analysis and infinite dimensional analysis, problems arising from other areas of mathematics such as real analysis, differential equations and differential geometry.

(2) Research fields related to Probability theory
Ergodic theory, dynamical system, stochastic control and mathematical finance.

We welcome visits and talks by many researchers from other universities, domestic and abroad.

Department of Mathematics

Dynamics and Fractals Seminar

Researchers and students working on various fields related to dynamical systems and fractals attend this seminar. We meet once a month for approximately 90 minutes. Each talk on his/her research is followed by discussions among all participants.

●COLLOQUIA

Department of Mathematics

Mathematics Colloquium

Colloquia take place on Monday afternoon at 16 : 30 in Room E404. They are directed toward a general mathematical audience. In particular, one of the functions of these Colloquia is to inform non-specialists and graduate students about recent trends, ideas and results in some area of mathematics, or closely related fields.